A Demand Responsive Bidding Mechanism with Price Elasticity Matrix in Wholesale Electricity Pools

by

Jiankang Wang

E.B., Zhejiang University, 2007

Submitted to the Department of Electrical Engineering and Computer Science in partial fulfillment of the requirements for the degree of
Master of Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2009

Author ....
Department of Electrical Engineering and Computer Science
May 21, 2009

Certified by ....
James L. Kirtley
Professor, Department of Electrical Engineering and Computer Science
Thesis Supervisor

Certified by ....
Scott Kennedy
Associate Professor, Water and Environment Program
Masdar Institute of Science and Technology
Thesis Supervisor

Accepted by ....
Terry Orlando
Chairman, Department Committee on Graduate Students

A Demand Responsive Bidding Mechanism with Price Elasticity Matrix in Wholesale Electricity Pools

by

Jiankang Wang

Submitted to the Department of Electrical Engineering and Computer Science
on May 21, 2009, in partial fulfillment of the
requirements for the degree of
Master of Science

Abstract

In the past several decades, many demand-side participation features have been applied in the electricity power systems. These features, such as distributed generation, on-site storage and demand response, add uncertainties to both the short-term and long-term operation of the modern power systems. On the contrary, many modern power systems are characterized by the deregulated market structure. How to operate these features under deregulated power markets is worth consideration.

This thesis presents a new demand responsive bidding mechanism in wholesale electricity pools. The proposed bidding mechanism models demand response with Price Elasticity Matrices (PEM). Under the proposed bidding mechanism, the resultant generation schedules and electricity rates become dependent variable on demand response. This relation gives bidding results that are closer to the actual market equilibrium. By applying this bidding mechanism, more efficient market behaviors are achieved in the short term, and generation and transmission resources are better utilizes in the long term. In addition, compared to the market clearing price and generation dispatch schedule settled by the traditional bidding mechanisms, bidding results obtained under our proposed mechanisms are more effective instructions for the design and implementation of demand-side participation programs.

This thesis presents the design of the proposed bidding mechanism in terms of its bidding rules, bidding acceptance rules and settlement rules. The bidding mechanism's mathematical model is formulated as an optimization problem. Bidding results are obtained as closed-formed solution of the optimization problem. In addition, this thesis presents an improved market interaction algorithm to implement the bidding mechanism. Multiple benefits of applying the bidding mechanism are shown by numerical example under various system statuses and end-user response types.

Thesis supervisor: James L. Kirtley
Title: Professor of Electrical Engineering

Acknowledgements

I would like to thank my thesis advisor, Professor James L. Kirtley for his guidance and support throughout the course of this thesis. I would like to thank Dr. Scott Kennedy, associate professor at Masdar Institute of Science and Technology, for his advisory work of this thesis. I enjoyed working with you both, and I appreciate your dedication to your students and to the field of engineering.

I want to thank all of the staff members and my fellow students from LEES who were always willing to help out. Special thanks to Yihui Qiu, Yehui Han, Riccardo Signorelli, Jing Ying Hu, Jae-Won George Hwang, Anthony Sagneri and Wei Li to their help in the lab, and to Vivian Mizuno for her support.

Thanks to my sponsor: The MIT Masdar Program.

Most importantly, I want to thank my family and especially my parents Qinghua Wang and Anning Xia, for their love and encouragement. I would not be where I am today if not for you.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Overview

During the past several decades, people have been constantly making effort to reduce the conflict between growing demand load and limited generation. To solve these problems, demand-side participation features, such as distributed generation, on-site storage and demand response programs, have been added to the modern power system [1-3]. Although the emergence of these features brings about more flexibility and options to both the supply and demand side, they also increase uncertainties in the power systems. For example, by sending certain price signals, some demand response programs can instruct end users to curtail, store or shift their potentially consumed electric energy. This operation may lead to temporary demand relief and price spike reduction. However, the resultant load profile of each time-period and the total energy to consume becomes different from the forecasted load value. Thus, in short-term power system operations like day-ahead electricity market, it is hard for system operators to predict the demand and to dispatch the generation for the next day; in long-term planning, planners may have difficulties to decide the capacity of the generation and transmission capacity.

So far, electricity market deregulation has been suggested as an effective measure for better utilizing generation and transmission resources as well as reducing electricity cost on both supply and demand sides. In the many deregulated electricity markets, bid-based auctions determine the electricity rate and generation schedules by solving optimization problems. This process makes the resultant electricity rate and generation schedules dependent variables on the forecasted demand. How to design and implement the mentioned features, such as demand response programs, under the complex deregulated market context is another problem in modern power systems [4-6].

The problem generated by the demand-side participation features and the deregulated market structure of modern power systems cannot be solved by only considering the demand-side participation or the electricity market separately [7, 8]. On one hand, among all these demand-side participation features, demand response programs have the most extensive effect on power systems and all end users. On the other hand, in the deregulated electricity markets, generation schedules and electricity rates are determined through bid-based auctions (which are also called bidding mechanisms). Therefore, a bidding mechanism considering demand-side market participation is a promising direction to solve the mentioned problems arising from both sides. However, the existing bidding mechanisms either ignore the demand-side or only cover limited form of the demand response programs (demand-side bid in emergency market), and fail to address the full mentioned features [8-10].

For this reason, this thesis presents a new demand responsive bidding mechanism in wholesale electricity pools. The proposed bidding mechanism models demand response with Price Elasticity Matrices (PEM). Under the proposed bidding mechanism, the resultant generation schedule and electricity rate become dependent variable on demand response. This relation gives bidding results that are closer to the actual market equilibrium. By applying this bidding mechanism, more efficient market behaviors are achieved in the short term and generation and transmission resources are better utilized in the long term. In addition, compared to the market clearing price and generation dispatch schedule settled by the traditional bidding mechanisms, bidding results obtained under our proposed mechanisms are more effective instructions for the design and implementation of the demand-side participation programs.

1.2 Outline and Contributions

This section includes an outline of the rest of the thesis. Chapter 2 gives the background of electricity markets, demand response and bidding mechanism designs. Section 2.1 introduces the concept of electricity market deregulation, the electricity market structures and architectures after deregulation. Section 2.2 introduces demand response's working principle and its role in the U.S. electricity markets. Furthermore, it states the problems and challenges that demand response brings to the U.S. electricity markets. Section 2.3 presents the designs of bidding mechanisms in electricity markets. It introduces the three parts of a bidding mechanism: bidding rules, bidding acceptance rules and settlement rules. Traditional bidding mechanisms in day-ahead market and real-time market are reviewed by these parts in this section.

After the background chapter, all the material presented is original, except for that which is repeated to show how it can be generalized by or compared to our new results. In Chapter 3 we cover those contributions which are of a theoretical nature.

- We propose a demand responsive bidding mechanism, which considers end-user response with inter-temporal load shifting. Bidding rules, bidding acceptance rules and settlement rules are defined for this bidding mechanism. PEMs are used to model all end-user response types. Bidding results obtained from the proposed bidding mechanism are closer to the actual market equilibrium.
- A mathematical model of the proposed bidding mechanism is formulated as an optimization problem. By deriving the closed-form solution of this problem, we show that the pricing structure of the proposed bidding mechanism is the same as the spot pricing structure proposed in Schweppe's work [11, 12]. In addition, we show that the shadow prices of the generation and demand inter-temporal conditions are included in the market clearing price under the proposed bidding mechanism.
- We give the sensitivity analysis of the proposed bidding mechanism under four disturbances: change in generation cost, changing in generation capacity, change in transmission limits and change in demand response programs. The robust conditions under these four disturbance types are derived.

- We give a full classification of the PEMs based on end-user response types. Furthermore, we point out the factors affecting the PEM's establishment: affecting periods, affected periods and incentive timings of demand response programs. We also present several methods of estimating the PEMs.
- Based on the market interaction algorithm proposed in David's work [13], we propose an improved algorithm for the proposed bidding mechanism. This improved algorithm can detect the two causes for market non-convergence: demand clears the market and steep local relative slope of the supply curve, where the second condition and the concept of *relative slope* is defined originally in this work. By applying this improved algorithm, market equilibriums are found in the previous non-convergent cases.

Chapter 4 presents numerical examples of the proposed bidding mechanism in day-head and hourly-ahead markets. Section 4.1 states the simulation environment and data background of the numerical examples. Section 4.2 and Section 4.3 give numerical examples under various end-user response types. Section 4.4 gives examples under system with generation-side contingencies. Section 4.5 gives examples under systems with renewable energy. Section 4.6 shows different non-convergence cases and how market equilibriums are found in these cases according to the proposed algorithm.

We close with Chapter 5 which summaries our results.

Chapter 2

Background

2.1 Electricity Market

An electricity market is a system for selling and purchasing electricity, using supply and demand to set the prices and schedules under given physical constraints. The design of an electricity market defines the physical dispatching procedure in the short term, and thus affects the power systems' planning decision in the long term. A basic understanding of electricity markets is essential for us to study the bidding mechanisms and the demand-side participation activities in the environment. This section introduces an important characteristic of modern electricity market deregulation. It also provides an overview of the structures and architectures of deregulated electricity markets.

2.1.1 Electricity Market Deregulation

History of electricity market deregulation

The concept of electricity market deregulation was first raised by Samuel Insull, who was elected president of the National Electric Light Association in 1898. In his historic presidential address to NELA, Insull explained not only why the electricity business was a "natural monopoly" but why it should be regulated and why this regulation should be at the state level, not the local level. Insull argued that

> *exclusive franchises should be coupled with the conditions of public control, requiring all charges for services fixed by public bodies to be based on cost plus a reasonable profit.*

These ideas led directly to regulatory laws passed by New York and Wisconsin in 1907. Unfortunately, the deregulation resultant from the action of these laws was unsuccessful. Both the end-users and the supply side agreed that competition was inefficient and that providing electricity was a natural monopoly.

Until the early 1980s, the energy market concepts and privatization to electric power systems was again brought up in Chile. The Chilean model was generally perceived as successful in bringing rationality and transparency to power pricing, but it suffered from the continuing dominance of several large incumbents and the attendant structural problems. Argentina improved on the Chilean model by imposing strict limits on market concentration and by improving the structure of payments to units held in reserve to assure system reliability. During the 1990s, the World Bank was active in introducing a variety of hybrid markets in other Latin American nations, including Peru, Brazil and Colombia, with limited success.

In 1990, the UK Government under Margaret Thatcher privatized the UK Electricity Supply Industry. The process followed by the British was then used as a model or at least a catalyst for the deregulation of several other Commonwealth countries, notably Australia and New Zealand, and regional markets such as Alberta. However, in many of these other instances the market deregulation occurred without the widespread privatization that characterized the UK example.

China dismantled the State Power Corporation on December 29, 2002 and set up 11 new companies in a move to end the corporation's monopoly of the power industry. The former State Power Corporation owned 46% of the country's electricity generation assets and 90% of the electricity supply assets. The new companies include two power grid operators, namely the State Power Grid and China South Power Grid. Each of the five electricity generation companies own less than 20% of China's market. They will compete with each other for signing contracts with the power grid operators. The country also set up the State Power Regulatory Commission on December 30, 2002, to supervise market competition and issue licenses to operators in the power industry [14].

Today, more than a dozen semi-deregulated electricity markets are operating in at least ten countries, with several operating in the United States, shown in figure 2.1.1. Between 1997 and 2007, the amount of competitive generation has increased almost five-fold, from 8.5% to 40% of the total U.S. generation capacity [10].

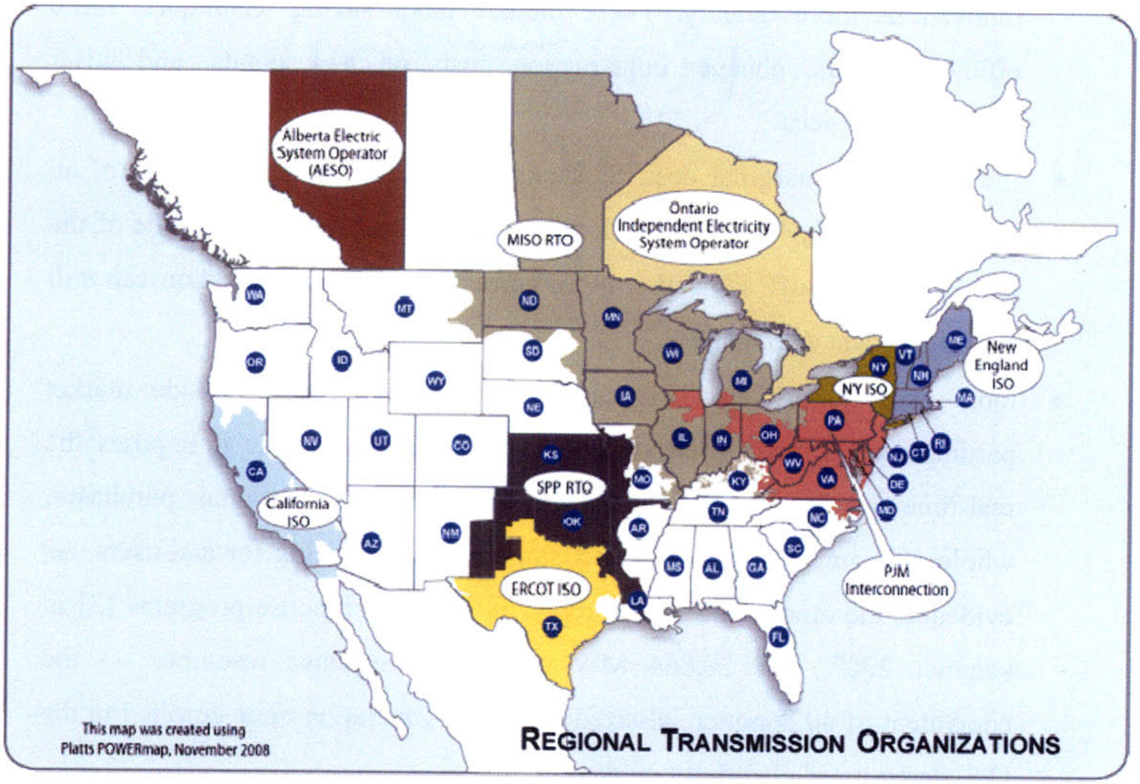


Fig. 2.1.1 The deregulation of the electricity markets in the U.S. The colored regions are the electricity markets that have been deregulated.

Benefits of electricity market deregulation

Until now, competition in wholesale power markets, as one stage of electricity market deregulation has been, and continues to be fostered by national policy. In each major energy bill over the last few decades, Congress has acted to open up the wholesale electric power market by facilitating entry of new generators to compete with traditional utilities. As the third major federal law enacted in the last 30 years to embrace wholesale competition, the Energy Policy Act of 2005 strengthened the legal framework for continuing wholesale competition as federal policy for this country. The Commission has acted quickly and strongly over the years to implement this national policy. The execution of these policies and the electricity market practice for many years has proved that multiple benefits can be achieved through electricity market deregulation. These benefits include [3, 15]:

- stronger *cost-minimizing* incentives than typical "cost-of-service" regulation. These incentives result in suppliers making many kinds of cost-saving innovations more quickly. These include labor saving techniques, more efficient repairs, cheaper construction costs on new plants, and wiser investment choices.
- lower price to marginal cost. Sioshansi [10] argues that this is less of an advantage simply because traditional regulation has stressed this side of the regulatory trade-off. However, more agree that price minimization can still be a significant advantage.
- more accurate pricing. This can be done through demand-side market participation (which is introduced in section 2.3). Because it imposes the real-time wholesale spot prices on the retailer's marginal purchases, wholesale competition should encourage real-time pricing for end-users. As evidence, the amount of load enrolled in demand-response programs [3] in summer 2007, was 20,864 MW of demand-response resources — the equivalent of 40 commercial-sized base-load generation units enrolled in the U.S. deregulated electricity markets.
- less pollution. Competitive electricity markets are not the single driver behind these regional differences, but the efficiency gains, wide geographic footprints and integration of renewable energies that accompany competition will become even more necessary with the approach of new climate change regulations. Restructured markets are clearly leading the way in addressing climate change, shown in figure 2.1.2.
- encouraging innovations. Distributed generation is an area in which innovations arise much more quickly under competition than under regulation; cogeneration is an example. Regulated utilities found such projects extremely awkward at best, so avoided them. A competitive market easily allows the flexibility that such projects require.
- getting both incentives of innovation and prices right at the same time. Again cogeneration provides an example. Once regulation decided to encourage it, they needed to price cogenerated power. A formula was designed with the

intuition of mimicking a market price. Naturally, political forces intervened and the result was long-term contracts signed at very high prices [15]. These gave strong, probably much too strong, incentives for cogeneration. A competitive market can get both incentives and prices right at the same time.

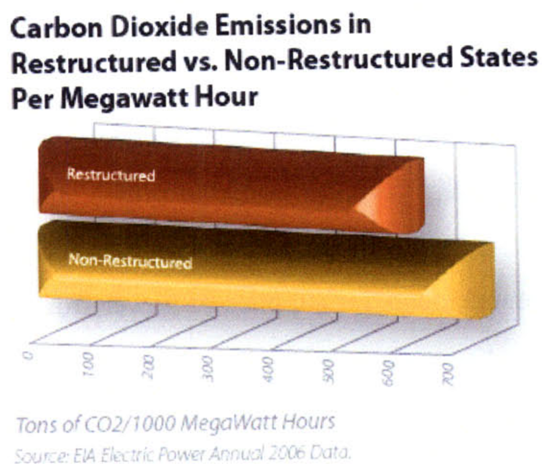


Fig. 2.1.2: Carbon dioxide emission in regulated and deregulated electricity market.[15]

2.1.2 Deregulated Electricity Market Structure

Participants in deregulated electricity markets

The electric power sectors in the United States can be described as generation, transmission, distribution and end-users, from where electricity is generated to where it is consumed.

The generation of electricity involves the creation of electric energy using falling water, internal combustion engines, steam turbines powered with steam produced with fossil fuels, nuclear fuel and various renewable fuels, wind driven turbines and photovoltaic technologies. Different types of generating plants are characterized by different shares of fixed and variable cost. This cost structure means there is an order for plant dispatch, the so-called "merit order", which minimizes total costs, bringing plant into operation as demand rises. Hence in a cost-based system, capacity with low variable and high fixed cost, such as nuclear, is operated as much as possible. This type of capacity is called base load. The reverse holds for the type of gas plants referred to above, which are operated at peak or intermediate load .

The transmission of electricity involves the use of wires, transformers and substation facilities to enable the high voltage "transportation" of electricity between generating sites and distribution centers. In regulated electricity markets, the transmission networks are owned by the utilities, who also own the generation and distribution sectors; in deregulated electricity markets, the transmission networks are owned by an Independent System Operator (ISO) or a Regional Transmission Organization (RTO) [8].

The distribution of electricity to end-users and local businesses at relatively low voltages relies on wires and transformers along and under streets and other rights of way. The distribution functions typically involves both the provision of the services of the distribution "wires" to end-users as well as a set of *retailing* functions. These retailing functions include making arrangements for supplies of power from generators, metering, billing and various demand management services.

End-users are residences and businesses who consume electricity. By their load profiles' characteristics, the end-users are usually classified as industrial, commercial and residential consumers.

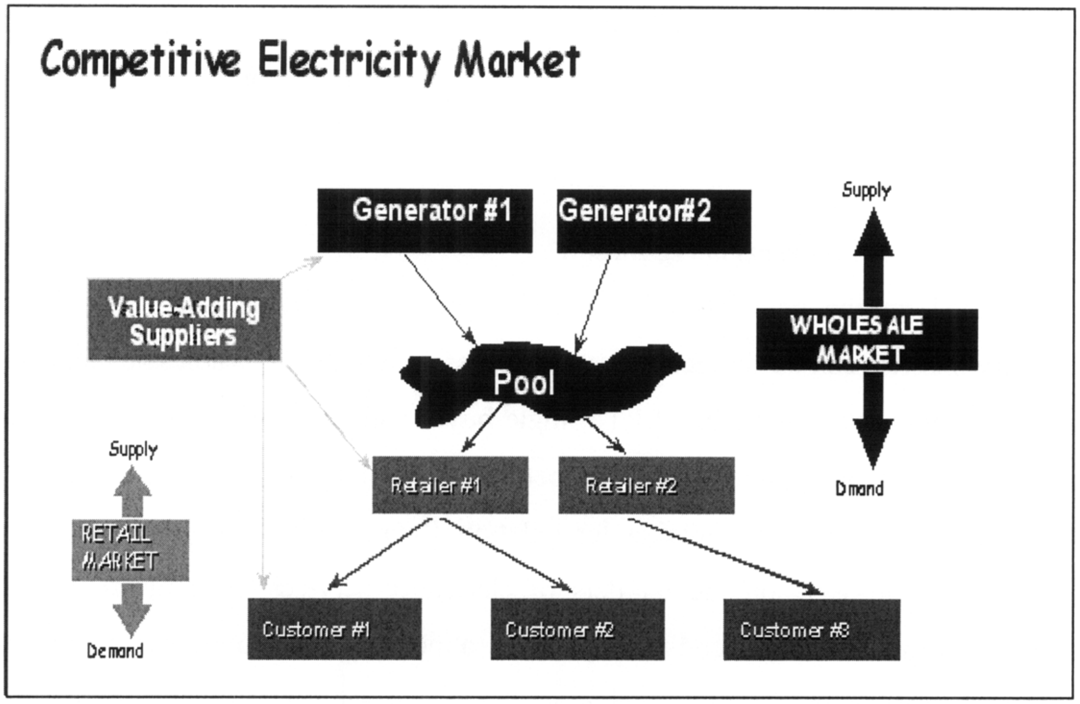


Fig. 2.1.3 Participants in deregulated electricity markets.[8]

Wholesale and retail electricity markets

Financially, after being generated, a single megawatt, is frequently bought and re-sold a number of times before finally being consumed. Those "re-sale" transactions make-up the retail electricity market; the other transactions, which are considered "sales for re-sale," make-up the wholesale electricity market, see figure 2.1.3.

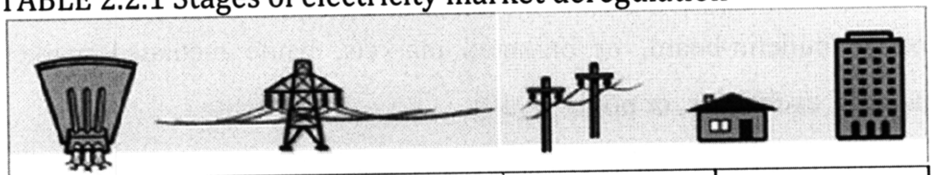
In the retail market, the retailers, who resale electricity, represent the suppliers. The end-users select the retailers for the electricity supply based on their services and prices. This market is out of the scope of this paper.

The wholesale market participants include suppliers who can generate power and connect to the grid, and retailers which are counterparties willing to buy the suppliers' output. The suppliers include utilities, independent power producers (IPPs), as well as some excess generation sold by traditional vertically integrated utilities. Retailers usually own the distribution wires connecting to the end-users. To be a participant in the wholesale market, however, one does not need to either own any generation or serve any end-use customers. Just as with many other commodities - pork bellies, oil or stocks - individual traders (or power marketers) exist who buy power on the open market and resell it. Depending on the market structures, an ISO or TRO is needed to run the transactions in the wholesale markets under given physical conditions of the power systems.

Stages of electricity markets deregulation

Based on the concepts of wholesale and retail markets, the deregulation of electricity markets can be divided into three stages, single buyer, wholesale competition, and retail competition, see table 2.1.1. Single buyer and wholesale competition exist in wholesale market deregulation; retail competition exists in retail market deregulation, and is the final stage of electricity market deregulation. Retail market deregulation allows free choices from end-users on the retailers of electricity services. In contrast, wholesale market deregulation requires end-users connect to the local distribution wires' owners. A number of regions in the U.S, including the Northeast, Mid-Atlantic, much of the Midwest, ERCOT and California allow for retail competition [10].

TABLE 2.2.1 Stages of electricity market deregulation



Market model	Monopoly	Single buyer	Wholesale competition	Retail competition
Definition	Monopoly through all sectors	Generation competition only	+ retailers competition	+ end-user competition
Generation competition?	No	Yes	Yes	Yes
Retailers competition?	No	No	Yes	Yes
End-user competition?	No	No	No	Yes

2.1.3 Deregulated Electricity Market Architecture

An electricity market's architecture is a map of its component "submarkets" given the "market type." Both the submarkets and the market type can be classified according to multiple categories.

Market Types

The most fundamental category of the market type is the transaction content. Based on this category, the market is either an energy market or a transmission-rights market. The transmission-rights market is better than an energy market in the sense that the system operator's role is minimized, and is refrained from trading or pricing energy. On the other hand, people think the transmission-rights market is impractical. A more practical way is that at least in real time, the system operator needs to buy and sell energy directly and needs to set different prices for energy provided at different locations.

Another basic category of the market type is based on the centralization degree of the market, see figure 2.1.4. There are two basic ways to arrange trades between buyers and sellers. One way is that buyers and sellers trade directly, that is, making a bilateral trade. The other way is that suppliers can sell their product to an intermediary who sells it to end-use customers. In order of increasing centralization, bilateral markets can be search, bulletin-board, or brokered markets, while mediated markets can be dealer markets, exchanges, or pools [9, 10].

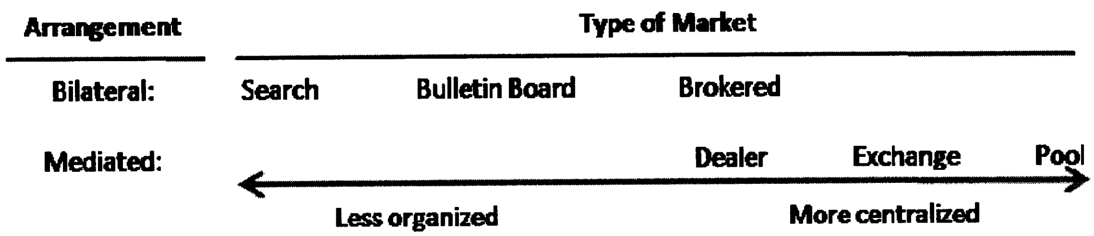


Fig. 2.1.4 Market types and centralization degree of the market.

In a *bilateral market* buyers and sellers trade directly. Such markets are flexible as the trading parties can specify any contract terms they desire, but negotiating and writing contracts is expensive. Assessing the credit worthiness of one's counter party is also expensive and risky.

In a *dealer market* dealers trade for their own account and usually maintain an inventory. Unlike the brokers in bilateral markets, the dealers buy the contract and hold it before reselling. There is no brokerage fee, but at any point in time the dealer buys for a price that is lower than the price he sells for. This difference is called the spread.

An *exchange* provides security for traders by acting as the counter party to all trades, eliminating the risk of creditworthiness. Exchanges that utilize auctions are called *auction markets*.

Pools are defined by the existence of side payments. In a pool, generators bid their marginal cost and certain other costs and limitation; whereas an exchange uses simple bids which express only an energy quantity and price. Pools accept some apparently losing bids. Accepted bids that would otherwise lose money are compensated with a "make-whole" side payment. Exchanges accept only bids that, according to their bid-in values, at least break even. Exchanges do not make side payments. Consequently, generators have to manipulate or "game" their bids in an exchange in some way to avoid a loss.

When designing an electricity market, it is hard to say if the bilateral market is absolutely better than the mediated market, or the other way around. The mediated market, due to its high centralization, can reduce trading costs, increase competition, and produce a publically observable price. On the other hand, the bilateral market, depending on design and circumstances, can inhibit collusion and generally provides more flexibility than the mediated market. Because of these characteristics of the bilateral and mediated market, the two market types are usually applied in different submarkets. The mediated market, for example the pool, is inherently a market for physical transactions, which are on a short timeframe such as real-time operations. This is because the pool is inflexible, and thus it can operate much faster than the bilateral market. In physical transactions, speed is crucial. Catastrophes can happen in seconds and system operators often need to exercise minute-by-minute control. Because of its speed, the pool can operate much nearer to real-time than the bilateral market. In contrast, the bilateral market (sometimes the less centralized exchange) is essentially forward markets for financial transactions. This is because with less centralization there is more room to earn commissions as brokers and to appropriate the spread as dealers. For markets of weeks and months in advance, physical deficiencies are inconsequential, and ordinarily they are settled at the subsequent spot price. This makes the bilateral market an obvious choice for forward markets. Both real-time markets and forward markets are defined by the concept of submarkets.

Submarkets

Submarkets can be classified by their spatial or temporal differences. In spatial classification, submarkets are a collection of multiproduct markets based on the fact that they are geographically distributed. When the transmission system is congested (or if losses are charged for, as they should be) electricity at location **A** is technically a different product from electricity at location **B** [12]. Consequently, an electricity market is a multiproduct, and every product defines a submarket.

On the other hand, in temporal classification, an electricity market includes forward markets, spot markets and ancillary market, see figure 2.1.5. Trading for the power delivered in any particular minute begins years in advance and continues until

Real Time (RT), the actual time at which the power flows out of a generator and into a load. All these markets, including the trade of electricity futures, except the RT market are classified as *forward markets*. Electricity futures typically cover a month of power delivered during on-peak hours and are sold up to a year or two in advance. Most informal forward trading stops about one day prior to real time. At that point, the system operator holds its Day-Ahead (DA) market. This is often followed by an hour-ahead market and a RT market. The RT market is the only submarket *spot market*. (Though some books use the term spot market to include DA and hour-ahead markets, this thesis only refers to RT markets by as spot markets.) Spot markets are physical markets, while forward markets are financial markets. This is because in spot markets all trades correspond to actual power flows; in forward markets, the delivery of power is optional and the seller's only real obligation is financial. In other words, if power purchased in forward market is not delivered, the supplier must purchase replacement power or pay liquidated damages.

Ancillary markets are submarkets that parallel with forward markets and spot markets,. Ancillary markets provide ancillary services in support of the basic service of generating real power and injecting it into the grid. Much more is needed to ensure that the supply of delivered power is reliable and of high quality. Some of these services in ancillary markets are indirect, but they are all concerned with dispatch, trade, and delivery of power. Organized by time scale, there are five types of ancillary submarkets. The services characterizing these submarkets are real-time balancing frequency and voltage, transmission security, economic dispatch, trading enforcement and the black-start capability. Among these services, economic dispatch is the only service in the time scale of this thesis' scope. Economic dispatch refers to using the right generators in the right amounts at the right times in order to minimized the total cost of production. We will give more details of economic dispatch in section 2.2.

The entire market

The definition of an entire market typically includes the market participant scope, submarkets as components, and the market type. The entire market of this thesis is wholesale DA and RT energy-trading electricity pools. The entire market is set so as it is a model or at least a catalyst in the United States. National policy for many years has been, and continues to be, to foster competition in wholesale power markets. As the third major federal law enacted in the last 30 years to embrace wholesale competition, the Energy Policy Act of 2005 strengthened the legal framework for continuing wholesale competition as federal policy for this country. The DA and RT markets represent forward and spot markets. Pools are inherently RT market type and are often used in DA markets.

Under this thesis's entire market definition, the FERC standard market design proposal describes the sequence of the energy markets' operations with the following structure [8]:

- First, the ISO undertakes a series of pre-day-ahead procedures and out-of-market actions to schedule generators with longer than one-day start-up or shut-down requirements.
- Second, the ISO operates a DA market that includes an auction with "security constrained unit commitment," which considers all the known transmission and generation unit constraints, within the limitation of the auction optimization.
- Third, the ISO takes several actions, for example, reliability unit commitment, to ensure reliability prior to real-time.
- Finally, in RT market, the ISO operate the power system through a power dispatch through a security constrained economic dispatch to determine auction market prices and supported also by physical dispatch instructions.

The bidding mechanisms in the above operations are described in section 2.3

2.2 Demand Response

Definition

During the past several decades, pressures to increase competition, reduce market power, improve reliability, and enable the use of cleaner renewable energy technologies have led to an increasing push for demand-side participation, in particular demand response programs, in competitive power markets.

Demand response refers to actions by customers that change their consumption (demand) of electric power in response to price signals, incentives, or directions from grid operators. In this report, Commission staff adopted the definition of "demand response" that was used by the U.S. Department of Energy (DOE) in its February 2006 report to Congress [3]:

> *Changes in electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized.*

This section will answer three questions on demand response: what role does demand response play in current power system of the United States; what are the working principles of demand response; and, what problems does demand response bring to the United States power system.

2.2.1 Demand Response's Role in the U.S. Power Systems

Motivation

A truly functioning electricity market incorporates dynamic supply and demand forces. A recently frequent criticism of current wholesale market designs has been that the demand-side of the market is not active; thereby creating the potential for supplier market power. Enabling demand-side responses as well as supply-side responses increases economic efficiency in electricity markets and improves system reliability. For this reason, demand response has been developed.

History

The use of demand response (DR) programs began with demand-side management (DSM) in the 1980s and early 1990s. The emergence and increase of DSM adoption was driven by a combination of a directive in the Public Utility Regulatory Policies Act of 1978 (PURPA) [3], and by state and federal regulatory and policy focus on DSM and integrated resource planning. At that time, the primary objective of most DSM programs was to provide cost-effective energy and capacity resources to help defer the need for new sources of power, including generating facilities, power purchases, and transmission and distribution capacity additions. DSM only refers to energy and load-shape modifying activities undertaken in response to utility-administered programs. Whereas DR includes all intentional modifications to consumption patterns of electricity of end-use customers that are intended to alter the timing, level of instantaneous demand, or the total electricity consumption [15]. With the recognition of existing wholesale markets' imperfection, DR becomes more important. Regulatory support and technical advances in controls, communications, and metering led to a marked increase in development of DR programs, such as load management, particularly direct load control programs and interruptible/curtailable service tariffs.

Benefits

Multiple benefits can be achieved by implementing DR programs in electricity markets:

- Reducing market power. A reduction in demand in response to a price spike is crucial to constrain the ability of suppliers to raise prices to inefficient levels (i.e., price levels inconsistent with consumer willingness or ability to pay), due to the exercise of market power or other anticompetitive behaviors.
- Maintaining resource adequacy. In markets where an "energy-only" approach is adopted to maintain resource adequacy, demand response may play an important role in maintaining a balance between supply and demand. This is particularly important in light of the cyclical nature of power plant construction activity. During the periods when a market is

left with inadequate reserve margins, demand response can provide an important backstop.

- Postponing long-term investment. It is sometimes argued that demand-side resources can be used to defer or displace transmission investments in either a regulated or a competitive market. Some encouraging programs have been launched in California, New York, and the Pacific Northwest region of the United States.
- Fostering reliability. Carefully crafted DR programs can be used to foster reliability in RT system operations. High wholesale prices or participation in programs through which loads curtail in response to instructions from an ISO in return for some financial compensation can assist the ISO in balancing supply and demand in RT and in managing reliability during emergency conditions.
- Managing risk. When viewed as a call option, DR may provide a variety of risk management benefits to an ISO or load-serving entity in a competitive market.

In addition, short-term financial benefits in DA and RT markets are observed by implementing DR programs. Not all consumers need to respond simultaneously for markets to benefit by lowered overall prices. One study suggested that shifting five to eight percent of consumption to off-peak hours and cutting another four to seven percent of peak demand could save utilities, businesses, and customers as much as $15 billion a year [16]. Another posited, "20 percent of customers account for 80 percent of price response." Others find that "only a fraction of all customers, perhaps as few as five percent, are needed to discipline electricity market prices." [17] In its comments to the Commission, the Demand Response and Advanced Metering Coalition (DRAM) said it "believes that demand response typically is capable of providing demand reductions of 3-5 percent of annual peak load for periods up to 100 hours or so per year." In California's statewide pricing pilot, 80 percent of load reduction came from 30 percent of customers [18].

Current status

Currently in the United States, there are regional differences in the use of demand response and how its use has changed over the past decade. Data collected from regional reliability councils and electric utilities by North American Electric Reliability Council (NERC) in its Energy Supply & Demand database provides a snapshot of regional potential and historical trends. Fig. 2.2.1 illustrates that Florida Reliability Coordinating Council (FRCC), Electric Reliability Council of Texas, Inc. (ERCOT), and the MidAmerican Power Pool (MAPP) had the largest percentage of demand response capability in 1998. It also shows that the amount of load management included in regional forecasts declined between 1998 and 2003. Regions with larger relative declines include ERCOT, Northeast Power Coordinating Council (NPCC), Mid-Atlantic Area Council (MAAC), and Western Systems Coordinating Council (WSCC). In 2003, due to the decline in capability in ERCOT and an increase in capability in Mid-America Interconnected Network (MAIN), the regions with the largest percentage capability are FRCC, MAIN, and MAPP [3].

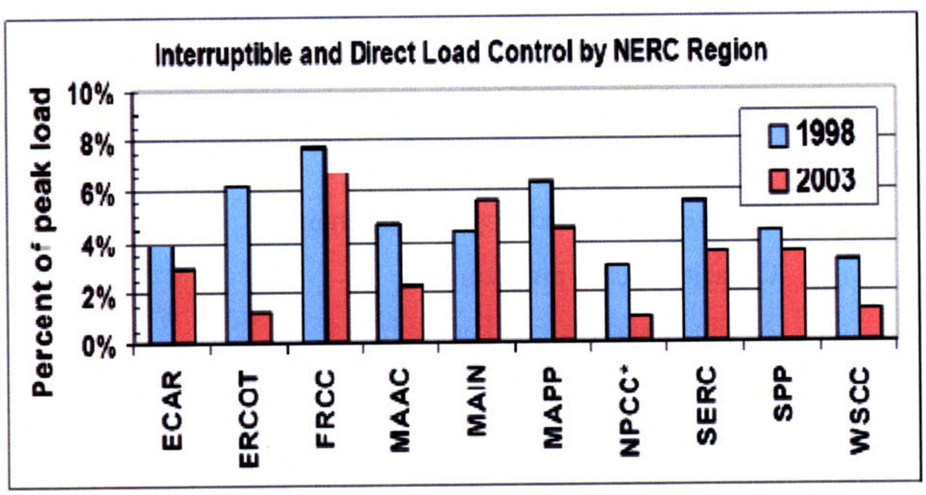


Fig. 2.2.1 Source: Data from NERC 1998 and 2003 summer assessments. NPCC* data is for 1998 and 2002 [15]

According to the literature on this issue, a contributing factor behind the decline shown in Figure 2.2.1 has been the waning of electric utility interest and investment in demand response over the past decade, due to changes in industry structure and the result of state electric restructuring plans. State and utility programs were dismantled in many restructured states that had previously supported extensive programs. In several states, such as Texas, load management was deemed a competitive service and regulated distribution companies were directed to divest their holdings [10]. In other states, utility divestiture of generation or transfer of the provider-of-last-resort (POLR) obligation removed a significant driver for utility investment by splitting up the benefits of demand response across multiple parties. Ample capacity reserves in many parts of the United States also contributed to declining utility interest and investment. Many states, such as Nevada, still support demand response and load management and operate integrated resource-planning programs that frequently include demand response and energy efficiency.

2.2.2 Demand Response's Working Principles

Classification

There are two primary categories of demand response: incentive-based demand response and time-based rates. Each category includes several major options:

- Incentive-based demand response
    - Direct load control
    - Interruptible/curtailable rates
    - Demand bidding/buyback programs
    - Emergency demand response programs
    - Capacity market programs
    - Ancillary-services market programs
- Time-based rates
    - Time-of-use
    - Critical-peak pricing
    - Real-time pricing

Incentive-based demand response programs offer payments for customers to reduce their electricity usage during periods of system need or stress. By adjusting or curtailing a production process, shifting load to off-peak periods, or running on-site distributed generation, customers can reduce the level of demand that they place on distribution networks and the electric grid. Customers who participate in incentive-based demand response programs either receive discounted retail rates or separate incentive payments. At a wholesale level, the impetus comes from independent system operators (ISOs) or regional transmission organizations (RTOs) and power marketeers. These programs can be triggered either for reliability or economic reasons. In the wholesale demand response programs, customer load reductions are aggregated by retailers, and then provided to the wholesale provider, such as an ISO, in exchange for an incentive.

The second type of demand response is comprised of time-based rates. A range of time-based rates are currently offered directly to retail customers; not all are time-varying, but they may promote customer demand response based on price signals. These are different from flat rates, which are unvarying and offer no price signals. Customer demand response, incentivized by time-varying price signals, is one way for electricity customers to move away from flat or averaged pricing and to promote more efficient markets.

Working principles

Both the incentive-based DR programs and time-based rates take effect through increasing elasticity of demand side by exposing end-users to incentives or time-varying pricing. Fig. 2.2.2 illustrates how this principle works and achieves the benefit of wholesale price reduction. Because generation cost increases exponentially near maximum generation capacity, a small reduction in demand will result in a big reduction in generation cost and in turn a reduction in price of electricity. In this example, the original demand curve is represented by a vertical line because it is assumed that the system is without DR programs. DR programs induce a negative slope on the original demand curve leading to small deduction in demand and a huge reduction in price.

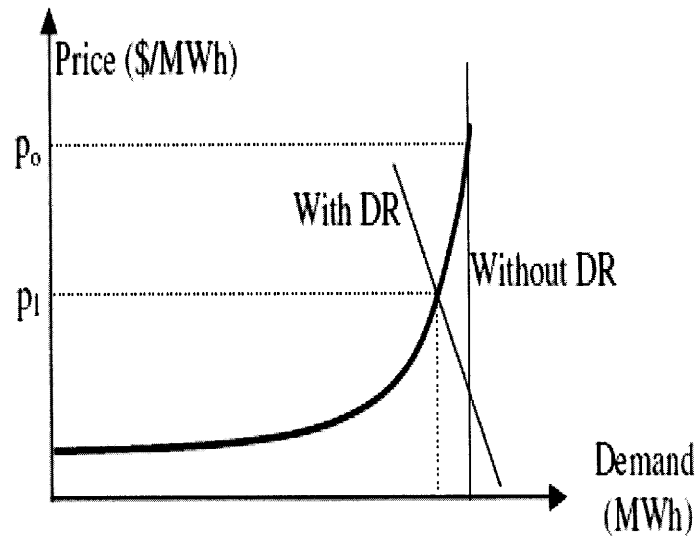


Fig. 2.2.2 a simplified illustration on how DR works by increasing demand elasticity

The two categories of demand response are highly interconnected and the various programs under each category can be designed to achieve complementary goals. For example, by adjusting customer load patterns or increasing price responsiveness, large-scale implementation of time-based rates can reduce the severity or frequency of price spikes and reserve shortages, thereby reducing the potential need for incentive-based programs. Care needs to be taken in their implementation to ensure that these programs do not work at cross-purposes.

The effect of either time-based rates or incentive-based demand response or incentive-based demand response on electricity sub-markets depends on the timeframe of the response. For example, real-time pricing or critical-peak pricing, which directly reflect wholesale prices, affect supply scheduling in day-ahead markets and during real-time dispatch. Time-of-use rates do not induce as rapid or as large of response. Incentive-based demand resources such as direct load control, can be used in ancillary markets, or as capacity resources in system planning.

The crux of demand response that this definition addresses is that it is an active response to prices or incentive payments. The changes in electricity use are designed to be short-term in nature, centered on critical hours during a day or year when demand is high or when reserve margins are low. Customer responses to high market prices can reduce consumption; this can shave wholesale market prices on a regular basis and thereby dampen the severity of price spikes in wholesale markets on extreme days. Customer response to incentives is an important tool available to operators of the electric grid to address reserve shortages, or retailers incorporate in their portfolios to match customer demand with available supply, and where available to individual customers to better manage their costs of doing business.

If changes in electricity prices last for a long time or are expected to do so, a longer-term price-based reduction in consumption through change in customer behavior may occur. Energy efficiency and conservation are often achieved while consumers are involved in demand response programs through actions taken by consumers to conserve their consumption of electricity during high price periods as they become more aware of their energy-usage patterns.

2.2.3 Demand Response's Problems and Challenges in the U.S. Power Systems

Although the emergence of demand response brings about more flexibility and options for both the supply and demand side, they also can introduce some complications into the operation of power systems. In the absence of a priority pricing scheme, where the prices at which load will curtail are announced to the market, the ISO may have difficulty understanding the slope or elasticity of the market's aggregate demand curve in a RT energy market. ISOs sometimes complain that demand response affects the accuracy of their near-term forecasts of demand, thus complicating the task of balancing demand and supply in real-time. If more generation is scheduled than needed, additional costs may be imposed upon the market [19, 20].

Thus, with demand response, in short-term power system operations, ISOs may find it harder to predict the future demand and commit or dispatch the correct amount of generation when there is an active demand response; while in long-term planning, planners may have difficulties deciding the needed generation and transmission capacity for load forecasts that include both short and long-term price elasticity of demand [16, 17].

On the other hand, successful demand response requires a correct combination of customer characteristics, economic incentives, metering and communications technology, policy support and most importantly market design. Existing market designs raise problems before and after the implementation of demand response programs: design and implementation of demand response programs can be more complicated due to lack of information in forward market; further difficulties arise in price settlement when the auction systems in DA and RT markets and demand pricing are designed independently [21].

In order for demand response programs to result in increased market efficiency, and not simply create additional uncertainty, it is critical that information regarding load behavior is provided to the market administrator and incorporated into the appropriate market price [22]. For this reason, demand-side bidding in day-ahead or real-time markets may have greater potential to increase efficiency than relying solely on a more passive demand response, where loads simply respond to real-time or forecasted prices.

2.3 Existing bidding mechanisms

All auctions solve some mathematical problems. Bids are submitted, some function of the bids is maximized or minimized, and the solution determines which bids are accepted. For example, say bids are submitted for the purchase of 100 candy bars. Each bid states a number of candy bars and a price. The auction problem may be to maximize the sum of the amount paid by each winning bid. The solution defines a set of accepted bids. There is one more step: settlement. The winners must pay and must receive the candy bars being auctioned.

The designs of auctions are called bidding mechanism. In auction theory, each bidding mechanism is specified by three sets of conditions: bidding rules, bidding acceptance rules, and settlement rules [10]. *Bidding rules* defines bidding participants, bids' content, and specific operation criteria. Restrictions on bidding have important consequences for the acceptance problem, and for market efficiency. *Bid acceptance and price determination* are usually lumped together because they are computed together, although conceptually they are distinct. On the other hand, *settlement* is not determined entirely by the solution to the acceptance problem but also uses separate price-determination rules, such as including penalties for noncompliance with commitments made in the auction. In our "candy bar" example, the settlement rules could specify that each accepted bid would pay as bid and receive the number of candy bars for, or it might specify that accepted bids would pay a price per candy bar equal to the lowest price per candy bar of any accepted bid.

Day-ahead (DA) and Real-Time (RT) markets run by Independent System Operators (ISO) that take the form of pools are operated as auctions. This section will focus on the bidding mechanisms of DA and RT auctions. Section 2.3.1 describes a typical bidding mechanism in DA markets. Section 2.3.2 presents alternative bidding mechanisms in DA markets. Bidding mechanisms in RT markets are given in section 2.3.3.

2.3.1 Auctions in Day-Ahead Pool Electricity Markets

Procedure

The description of DA markets often focuses on presenting an auction problem. The auction problem takes place in three stages,

1. Bids are submitted.
2. Some bids are accepted and prices are determined.
3. Accepted bids are settled at the determined prices.

A general bidding rule

Corresponding to these three stages, in auction theory, each bidding mechanism is specified by three sets of conditions: bidding rules, bidding acceptance rules, and settlement rules. In a DA pool, the bidding rules typically define the auction as sealed-bid with multipart bids. Suppliers submit bids prior to a trading deadline (usually the prior morning for the day-ahead and about 1 hour before the real-time market). These bids must usually takes the form of equation (1) [10]

$$B^G(P) = < P_{max}(t), P_{min}(t), C(P), C_{st}, C_{fix}, P_{rpu}, P_{rpd}, t_{on}, t_{off}, z > \quad , \qquad (1)$$

where

$B^G(P)$	bid submitted by supplier S with power profile P;
$P_{max}(t)$	maximum MW available at hour t;
$P_{min}(t)$	minimum MW at hour t if the generator selected online;
$C(P)$	cost function (\$/MWh) over the range of available output;
C_{st}	start up cost of the generation unit (\$);
C_{fix}	fix cost of the generation unit (\$);
P_{rpu}	ramp-up limit of the generation unit (MW/h);
P_{rpd}	ramp-down limit of the generation unit (MW/h);
t_{on}	minimum up time of the generation unit (h);
t_{off}	minimum down time of the generation unit (h);
z	other bids or constrains of the generation unit, such as reactive power.

A general auction formulation

After receiving the bids from suppliers, ISO conducts "security constrained unit commitment," in which "security" enforces the balance between supply and demand in the system (and probably on every node of the system). The unit commitment specifies exactly which generation units should be turned on in each hour, their level of output, and the length of time they should run over the day, based on start-up and energy bid prices and the other financial and physical parameters and transmission network constraints. Thus in a DA pool, bidding rules require a multipart bid from one generator per day to conduct unit commitment. On the contrary, in exchange markets, bidding rules require 24 hourly bids of one generator containing only energy quantity and prices. Moreover, in exchanges instead of unit commitment economic dispatch is performed, which can be formulated as a linear programming problem.

Mathematically, the security constrained unit commitment is done by solving a mixed-integer optimization problem with the objective of minimizing total generation cost as defined by the bids. A general form is shown in (2) [10],

$$\min \sum_{j}^{NG} \sum_{t}^{T} C^{j}\left(P^{j}(t), u^{j}(t)\right), \forall j \in \mathcal{G}, i \in \mathcal{D}$$

$$\text{s.t.} \quad L\left(P^{j}(t), u^{j}(t), P^{i}(t)\right) = 0, \forall t$$

$$G\left(P^{j}(t), u^{j}(t)\right) \leq 0, \forall j, t$$

$$Z_{k}\left(P^{j}(t), u^{j}(t), P^{i}(t)\right) \leq 0, \forall t$$

where

where	$\mathcal{G}$	set of generation units;
	$\mathcal{D}$	set of load units;
	NG	number of generation units in set $\mathcal{G}, = \lvert\mathcal{G}\rvert$;
	T	transaction time horizon of DA markets;
	$P^{j}(t)$	active power of generation unit j at hour t;
	$P^{i}(t)$	active power of end user i at hour t;
	$u^{j}(t)$	unit commitment variable of generation unit j at hour t,

$$= \begin{cases} 1, & \text{if committed} \\ 0, & \text{if decommitted} \end{cases}.$$

$u^j(t)$ and $P^j(t)$ are decision variables for generation unit j's commitment and active power output at hour t. $P^i(t)$ is a parameter of ISO forecasting on load unit i at hour t.

The objective function $\min \sum_j^{NG} \sum_t^T C^j\left(P^j(t), u^j(t)\right), \forall j \in \mathcal{G}, i \in \mathcal{D}$ states by choosing $u^j(t)$ and $P^j(t)$ to minimize the total generation cost of NG units during T, which is a typical form in demand bundled bidding mechanisms. Alternative forms of the objective function are applied under various bidding acceptance rules.

Constraint $L\left(P^j(t), u^j(t), P^i(t)\right) = 0, \forall t$ imposes system balance condition between electricity generation and load. The specific forms of this constraint depend on settlement rules of the bidding mechanisms in DA markets.

Constraint $G\left(P^j(t), u^j(t)\right) \leq 0, \forall j, t$ is a general form of generation unit j's limits. In practice, such constraint may be maximum and minimum power output, ramp-up and ramp-down limits, least online and offline time. The specific forms of this constraint depend on the bidding rules of bidding mechanisms in DA markets.

Constraint $Z_k\left(P^j(t), u^j(t), P^i(t)\right) \leq 0, \forall t$ is a general form of transmission network limits. Depending on the accuracy requirement and ISO's computation capability, this constraint can be nonlinear or approximated to linear. Under some specific settlement rules, say uniform payment, this constraint can be ignored [11]

By observing the above general formulation, we conclude that the three sets of conditions of a bidding mechanism define different parts of the unit commitment optimization problem. Bidding rules define the decision variables, parameters, and generation operation constraints, bidding acceptance rules define the objective function to optimize, price settlement rules formulate the other constraints that the objective function is subject to. Based on this conclusion, we will further discuss the alternative bidding mechanisms in DA markets by looking at their optimization formulations in the next section.

2.2.2 Alternative bidding mechanisms in DA markets

Alternative bidding mechanisms characterized by their three sets of conditions exist in DA markets. By looking at their bidding rules, bidding acceptance rules and price settlement rules, we exam these bidding mechanisms in this section.

Bidding rules

In a DA pool, the bidding rules require that suppliers submit multipart bids taking the form of (1). These bids must specify the minimum and maximum MW that can be produced by the generator, the price of energy ($/MW) over the range of its available output, a start-up cost ($), a no-load cost ($), and a number of physical characteristics, such as how rapidly the generator can increase or decrease output (called the "ramp rate" and measured in MW/hour). Markets with a bidding mechanism of which bidding rules require bids only from supply side is called demand bundled markets, such as presented in the general form (2). Demand bundled markets have multiple disadvantages as stated in section 2.2. To overcome these disadvantages, several demand unbundled bidding mechanisms have been developed in recent years [10].

Emergency Demand-Side Bidding (EDSB) programs are a bidding mechanism that allows participation from the demand side. This kind of programs is designed to reduce power usage through the voluntary shutting down of businesses and large power users. Usually, bidding rules in this bidding mechanism require demand bids stating the MW they will reduce and the price of reduction ($/MW). Companies, mostly industrial and commercial, sign up to take part in the programs. Once an emergency happens in real time, the ISO calls the accepted demand bids in DA markets. This action realizes the financial bids into physical load reduction. Payments are made based on the demand bids' real-time performance and real-time market prices. A practical example of this bidding mechanism is the NYISO's Day-Ahead Demand Response Program (DADRP). DADRP allows energy users to bid their load reductions, or "negawatts", into the Day-Ahead energy market as generators do. Offers determined to be economic are paid at the market clearing price [8].

Another demand unbundled bidding mechanism implements the opposite concept of the emergency demand-side bidding programs. In wholesale competition stage of electricity market deregulation, retailers representing demand side are allowed to participate in market transactions. Thus, in DA markets, bidding rules require retailers, instead of submitting the energy to reduce in the 24 hours, to submit bids with MW to consume, denoted in (3),

$$B^D(P,t) = < P(t), p(P,t) >,$$

where	$B^D(P,t)$	bids submitted by retailer D with consumption profile P and hour t;
	P(t)	consumption active power profile (MW);
	p(P,t)	price function on power P and hour t (MW/$);

p(P,t) is a function that depends on both power, P, and consumption hour, t, which is due to the reason that consumer utility of electricity is sensitive to t.

This bidding rule is raised in FECR's Standard Market Design Proposal (2006), and is the current implemented bidding rule in wholesale competitive markets. Notice that comparing to suppliers' bids, $B^G(P)$, retailers' bids take the form of $B^D(P,t)$. In other words, in DA markets, suppliers submits one bid for the next 24 hours, while retailers submit 24 single bids for the next day. For this reason, this bidding mechanism is also called single hourly bid (SHB).

The existing EDSB and SHB have several deficiencies. Firstly, in wholesale competitive markets, forecasting day-head demand rather than allowing retailer-participating auction will inhibit market equilibrium's allocation and consequently cause inefficient market operation. Secondly, EDSB limits demand-side participation to emergency demand response programs. Many other demand response programs with the objective of system adjustment, such as real-time pricing programs, are left out in EDSB's design. Thirdly, SHB ignores loads' operation constraints and inter-temporal shiftability. In other words, the energy consumption of one hour may affect the energy consumption of the other 23 hours. For example, some loads need continuous operation, thus their power consumption at a certain hour is coupled with the nearby hours. In addition, for SHB, previous works shows that allowing 24 hourly bids instead of a single bid for the day can increase market power [10].

To improve the stated deficiencies, Su and Kirschen [21] propose a demand-side bidding mechanism considering regular inter-temporal demand response. The bidding rules of this bidding mechanism takes the form shown in (4)

$$B^{D}(P) = < P(t), p(P,t), P_{rp}, E_{max}, >, \tag{4}$$

where

where	$B^{D}(P)$	bids submitted by retailer D with consumption profile P;
	P(t)	consumption active power profile (MW);
	p(P,t)	price function on power P and hour t (MW/$);
	P_{rp}	load ramp rate (MW/h);
	E_{max}	maximum energy of loads under retailer D to consume over the next 24 hours (MW·h).

$B^{D}(P)$ represents one bid for the 24 hours instead of 24 bids in SHB. P_{rp} and E_{max} impose inter-temporal constraints on loads consumption. However, these constraints only describe the load shifting within the close hours rather cross the whole auction period. In addition, the information required from demand bids is usually too discrete to estimate. If there are 500 different loads under a retailer, $B^{D}(t)$ will have a huge size.

Bidding acceptance rules

Bidding acceptance rules define the objective function of the unit commitment problem. In demand bundled DA markets, the bidding acceptance rule is to minimize the total generation cost over the next 24 hours. Under the bidding rules stated in section 2.3.1, the generation cost is the sum of electricity production cost, no-load cost and start-up cost. In demand unbundled DA markets, the same bidding acceptance rules apply to EDSB, since the emergency responding loads can be perceived as generation units. Thus the acceptance rule of EDSB is to minimize the total of generation and emergency responding demand. For SHB and other regular demand-side participation bidding mechanisms in wholesale competitive markets, the auctions will maximize actual total surplus [23].

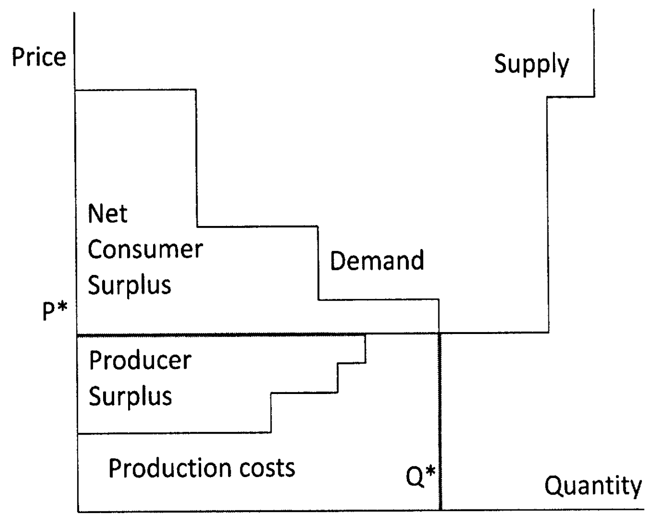


Fig. 2.3.1: Net consumer surplus, producer surplus and social welfare in market transactions.

Total surplus is the sum of (net) consumer and producer surplus, and it is also the gross consumer surplus minus production costs. Inheriting the notations of general form (2), the objective is $\max \sum_t(\sum_i p_i(P(t)) - \sum_j C_j(P(t)))$. From Fig. 2.3.1, we can observe that in an unconstrained system, total surplus can be maximized by turning the demand bids into a demand curve and the supply bids into a supply curve and finding the point of intersection (P*, Q*), which is called a market equilibrium. Under a market equilibrium, both the supplier and demand reach a stable status. This gives both the market price and a complete list of the accepted supply and demand bids. Unfortunately, transmission constraints and constraints on generation output and load consumption (as in (1) and (2)) can make this selection of bids infeasible. In this case it is necessary to try other selections until a set of bids is found that maximizes total surplus and is feasible.

Settlement rules

Bidding acceptance rules determines what bids are accepted, or partially accepted, and this is determined through simple accounting of the quantities bought and sold. Prices are determined by a separated set of settlement rules. Since forward markets are only financially effective, prices determined in these markets are not the final prices if considering spot markets. The payments that generation units receive depend on their real-time physical output in addition to the amount received in the DA market. For this reason, settlement rules in DA markets only partially determine suppliers' payments. In addition, we make the assumption that market equilibriums can be found for simplicity. In this case, the buying prices and selling prices are identical for the generation units and retailers at the same time and location.

A most fundamental type of settlement rules is uniform payment. Market prices are set by specifying the balance condition in (2) as total generation output equals to total load consumption as (5)

$$\sum_j P^j(t) - \sum_i P^i(t) = 0, \forall t \in \mathcal{T}, j \in \mathcal{G}, i \in \mathcal{D}. \qquad (5)$$

Applying the Karush-Kuhn- Tucker (KKT) conditions to the obtained problem (2), we get the dual variable associated with (5) at the optimum as the price vector of all generation units. The price vector has all its entries of the same value, commonly denoted as λ. The physical meaning of λ is the system's marginal generation cost, meaning the total generation cost ($) raised by increasing one MW demand.

Uniform payment is feasible only when the transmission network has sufficient capacity. However, congestion prohibits low-cost generation units from supplying loads on the other end of congested transmission lines, and makes uniform payment unfair [12]. To solve this problem, nodal pricing is developed. In addition to (5), transmission constraints (6) are imposed.

$$\sum_{j\in\mathcal{G}_b} P_j(t) - \sum_{i\in\mathcal{D}_b} P_i(t) - \sum_{n} P_{b,n}(t) = 0\ \forall t \in \mathcal{T}, \forall b \in \mathcal{B}, \forall\, b,n \in \mathcal{N} \quad (6)$$

$$-P_{b,n}^{max} \le P_{b,n}(t) \le P_{b,n}^{max}, \forall b,n \in \mathcal{N}, \forall t \in \mathcal{T},$$

where $\mathcal{B}$ set of indices of network buses;

$\mathcal{N}$ set of indices of transmission lines;

$\mathcal{G}_b$ set of generation units at bus b;

$\mathcal{D}_b$ set of loads at bus b;

$P_{b,n}(t)$ power transmit from bus b to bus n (MW);

$P_{b,n}^{max}(t)$ transmission capacity from bus b to bus n (MW);

$P_{b,n}(t)$ is defined by other parameters of the transmission system and its physical condition, which are not stated here.

Applying the KKT to the obtained problem (2), we get the sum of dual variables associating with (5) and (6) in optimum as the price vector of all generation units. The price vector's entries are composed of system marginal cost λ and the dual variable of (6), commonly denoted as μ_b. The physical meaning of λ plus μ_b is system's nodal marginal generation cost, meaning the total generation cost ($) raised by increasing one MW demand at node b.

Further improvements can be made on nodal pricing, if we consider the transmission losses in price settlement. Rewrite (5) as (7),

$$\textstyle\sum_j P^j(t) - \sum_i P^i(t) - \sum_{n,b} P_{n,b}^L(t) = 0, \forall t \in \mathcal{T}, j \in \mathcal{G}, i \in \mathcal{D}, n, b \in \mathcal{N} \qquad (7)$$

where $P_{b,n}^L(t)$ is the transmission losses from bus b to bus n. $P_{b,n}^L(t)$ is defined by other parameters of transmission system and physical condition, which are not stated here. Applying KKT condition to the optimization problem with (7), we derive a price vector from the sum of dual variables of (6) and (7). The obtained dual variables associating with (7) are composed of system λ and a losses cost term, while the dual variables of (7) remain as μ_b. The prices' physical meaning is system's nodal marginal cost under transmission losses.

In practice, a centralized pool market uses "make-whole" side payments to, in effect, pay different prices to different suppliers at the same time and location. These payments are only made when an accepted supplier would lose money at its as-bid costs given the pool price. In other words, the price paid to the a generation unit in a DA pool equals to any of the above price plus a side payment covering its commitment lost [12].

The mentioned three settlement rules are *ante post* price settlement, which partially determine price before dispatch. Other type of settlement rules, such as *ex post* price settlement, will be discussed in the next section.

2.3.3 Settlement Rules in Real-Time Markets

The RT market is a physical market, as all trades correspond to actual power flows. Unlike a DA pool, a RT pool cannot use bids for lack of transaction time. Though not holding auctions, RT markets determine RT prices and final payments for suppliers. In other words, price settlements of RT and DA markets together define settlement rules of a bidding mechanism of the two markets. For this reason, price settlement in RT markets can be thought as the second stage of its corresponding DA bidding mechanism .

Price settlement in RT markets is *ex post* pricing, meaning price determined after power generation. As forward markets, DA markets can be financially effective or ineffective. Based on this criterion, RT settlements generally fall into two types: two-settlement system and post-settlement system.

Two-settlement system

The concept of two-settlement system is to establish penalties for noncompliance with commitments made in the DA auction. Under two-settlement system, an RT pool works like a classical Walrasian auction: a price is announced and suppliers and consumers respond. In a RT market, trades are not under contract: power that just shows up, or is taken, in real time and suppliers or retailers accept the spot price. The difference is that in a power market trading takes place all the time; there is no waiting to trade until the right price is discovered. The RT price is determined by total actual (RT) supply and demand. ISOs adjust the output of already committed units on a 5-15 minute basis [10]. The adjustment procedure usually involves multiple ancillary markets, and finally settle at an RT price p_0. If a supplier sells P_1 to the ISO in the DA market for a price of p_1 and then delivers P_0 to the RT market, it will be paid:

$$\text{Supplier is paid: } P_1 \times (p_1 - p_0) + P_0 \times p_0 \,. \quad (8)$$

The incentive of this settlement rule are revealed by (8). When real time arrives, p_1 and P_1 have been determined in the DA market. Assuming the market is competitive, suppliers will also take p_0 as given, so by real time, the entire first term will be viewed as a "sunk" cost or an assured revenue. This leaves the second term as the only one that can provide an RT incentive for generator behavior, and this term pays the generator the RT price for every MW produced. Consequently the generator will behave exactly as if it is selling all of its product in the RT market. This can be proven by considering the supplier's profit, which is revenue minus cost, and the profit it would have had if it traded only in the RT market. Consider a simple illustration: suppose more power is needed and there is a generator with a DA contract and a marginal cost of $65/MWh while the RT price is $60/MWh. In a competitive pool, that generator would not produce even if it had sold its power in the DA market. It would earn more by buying RT replacement power than by generating.

Post settlement system

Post-settlement system uses another concept: penalties for noncompliance with the optimal generation point in the RT market. In the RT market, no price signals or generation instructions are given from the ISO under this settlement system. If generation unit j is measured as delivering P_0^j to the RT market afterwards, the ISO formulates an optimization problem shown as (9)

$$\min \textstyle\sum_j C_j(P^j) \;, \forall j \in \mathcal{G} \quad (9)$$
$$\text{s.t.} \sum_j (P^j - P_0^j) = 0$$

$$Z_k(P^j, P_0^j) \leq 0, \forall j \in \mathcal{G}$$
$$0 \leq P^j \leq P_0^j + \epsilon, \forall j \in \mathcal{G},$$

where ϵ is a small positive number.

Constraint $\sum_j (P^j - P_0^j) = 0$ is the transformed balance condition; Constraints $Z_k(P^j, P_0^j) \leq 0, \forall j \in \mathcal{G}$ are the transformed network limits; The last constraints, $0 \leq P^j \leq P_0^j + \epsilon, \forall j \in \mathcal{G}$, are the limits on deviation of the optimal generation point. The solution to (9) is considered as the optimal generation point that minimizes the total generation cost during the concerned real-time period. Dual variable of the balance condition under the optimal solution is the final price that suppliers will receive. The physical meaning of the settled prices is the marginal cost of deviation from the optimal generation point.

Under post-settlement system, DA auctions only select the units to commitment in the next 24 hours, but bids are financially ineffective. RT markets adopting this settlement rule are good examples that decouple bidding acceptance rules and settlement rules in a bidding mechanism. This settlement rule is currently only applied to supply side in RT markets. In other words, demand deviation from the optimal operation point will not cause any penalties.

The RT price should be set by taking into account the full supply and demand response. If this is not done, it will be necessary for the system operator to circumvent the market in some way to balance supply and demand.

Chapter 3

A Demand Responsive Bidding Mechanism with Price Elasticity Matrix

The emergence of Demand Response (DR) programs brings about more flexibility and options for both the supply and demand side. However, they also increase the uncertainties of power systems in short-term operation and in long-term planning. Moreover, the power systems, which are operated and planned without considering DR programs, also bring inaccuracies to the design and implementations of DR programs.

In order to get information of DR into electricity markets, some previous works proposed bidding mechanisms that allow demand-side participation [5, 21]. However, these bidding mechanisms cover only a limited form of demand response (demand-side bid in emergency market), and fail to exploit the full potential of demand-side resources. This thesis addresses this challenge by proposing a new demand-side bidding mechanism, in which demand-side participation is modeled with Price Elasticity Matrices (PEM) [24]. This feature allows the demand-side participant to specify inter-temporal constraints, which is not possible under current demand-side bidding mechanisms.

For illustration simplicity, this thesis presents the proposed demand responsive bidding mechanism under the entire market of a wholesale DA and RT energy-trading electricity pool. In this thesis, the DA market is hourly-based and its auction period is 24 hours; the auction period of the RT market is 5 minutes. With a few modifications, the proposed bidding mechanism can be applied to an extensive scope of wholesale competitive markets.

This chapter gives the theoretic part and main mathematical conclusions of the proposed demand responsive bidding mechanism. Section 3.1 and Section 3.2 describes the bidding mechanism in terms of the three sets of conditions: bidding rules, bidding acceptance rules and settlement rules. Section 3.3 addresses specific issues on PEM. Section 3.4 gives the algorithms of operation the proposed bidding mechanism.

3.1 The bidding rules

The bidding rules of the proposed bidding mechanism have the same general frame of traditional DA auction. It adopts general bidding chronologically and requires sealed multipart bids. Moreover, the supplier's bid profile takes the same form of (1) in the proposed bidding mechanism [23].

$$\mathrm{B^G(P)} = < \mathrm{P_{max}(t), P_{min}(t), C(P), C_{st}, C_{nld}, P_{rpu}, P_{rpd}, t_{on}, t_{off}, z} > , \qquad (1)$$

The differences between traditional bidding mechanisms and the proposed demand responsive bidding mechanism exist in retailers' bid profile, presented in (10).

$$B^D(P, \varepsilon_{T\times T}) = < P_0(t), P_{ref}(t), p_{ref}(t), P_{max}(t), P_{min}(t), \varepsilon_{T\times T} > \quad (10)$$

where	$B^D(P, \varepsilon_{T\times T})$	bids submitted by retailer D with consumption profile P and PEM $\varepsilon_{T\times T}$;
	$P_0(t)$	initial value of consumption active power at t (MW);
	$P_{ref}(t)$	reference consumption active power at hour t (MW);
	$p_{ref}(t)$	reference selling price of retailer D at hour t ($/MWh);
	$P_{max}(t)$	maximum consumption active power at hour t (MW);
	$P_{\min}(t)$	maximum consumption active power at hour t (MW);
	$\varepsilon_{T\times T}$	PEM of retailer D within a timeframe of length *T*.

Profile (10) essentially approximates a multi-dimensional demand curve, which states the self price-demand relation as well as cross price-demand relation. Two key assumptions are required for this approximation. They are

- the total consumption quantity submitted by the retailer is much larger than the consumption pattern of each load. Without this assumption, the demand curve is constructed by energy blocks, and thus is not continuous and differentiable;
- the demand curve can be linearized around the reference points of price and demand.

Figure 3.1.1 illustrates (10) on a two-dimensional demand curve of a single hour t. The linear demand curve, $P(p(t))$, can be described by its slope, ε, and a reference point on the curve, $(P_{ref}(t), p_{ref}(t))$ as in (11),

$$P(t) - P_{ref}(t) = \varepsilon\big(p(t) - p_{ref}(t)\big) \quad (11)$$

where the slope $\varepsilon = \frac{dP(t)}{dp(t)}$. The reference points can be any known points on the demand curve. The two limits, $P_{max}(t)$ and $P_{min}(t)$ specify the feasible region of the demand curve. When the proposed bidding mechanism is applied to multiple retailers over T hours, equation (11) will be calculated for iterations to search for the market equilibrium. (The detailed algorithm is introduced in Section 3.4.) For this reason, an initial point $P_0(t)$ is submitted for the searching. The value of this initial point won't affect the final market equilibrium. However, depending on the searching algorithm, the selection of $P_0(t)$ will affect the convergence time and even other searching performances.

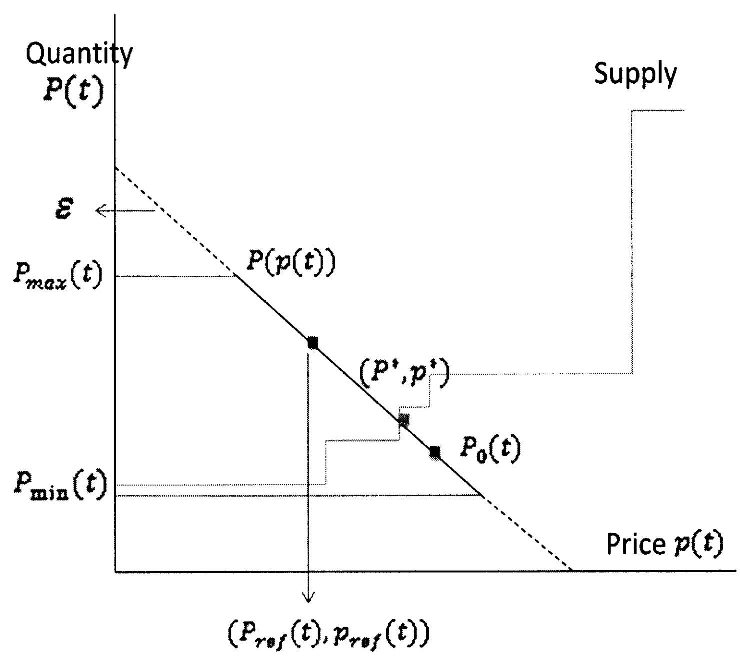


Fig. 3.1.1 Searching for the market equilibrium with elasticity, reference points and operation limits.

For demand curves over several hours, T, the slope ε changes from a scalar to a price elasticity matrix, $\varepsilon_{T\times T}$, where T is the PEM's dimension. The time period of interest, T, depends on the purpose of the analysis and could be determined by the market structure or the generation forecasting capability. Since this thesis presents the bidding mechanism in an hourly-based DA market, T is set as 24. The PEM's elements is defined in (12)

$$\tilde{\epsilon}_{t\tau} = \frac{\partial P_t / P_t}{\partial p_\tau / p_\tau}, \qquad (12)$$

where P_t is the quantity of electricity consumed in hour t, and p_τ is the price during hour τ. When perturbations are small, P_t/p_τ can be linearized around a fixed reference value, and the *normalized* elasticity is simply the partial derivative of the electricity demand with respect to the hourly price. The normalized price elasticity is,

$$\epsilon_{t\tau} = \frac{\partial P_t}{\partial p_\tau}, \qquad (14)$$

When $t = \tau$ in (14), the elasticity is defined as the *own-elasticity*, representing a demand change in response to a price change in the time period; when $t \neq \tau$, the elasticity is defined as the *cross-elasticity*, representing a demand change due to a price variation over any other time period. A vector representation can be used to calculate the total demand deviation over a certain time period:

$$\Delta P = \varepsilon_{T\times T}\Delta p$$

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_T \end{bmatrix} = \begin{bmatrix} \epsilon_{11} & \epsilon_{11} & \cdots & \epsilon_{1T} \\ \epsilon_{21} & \epsilon_{22} & \cdots & \epsilon_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon_{T1} & \epsilon_{T2} & \cdots & \epsilon_{TT} \end{bmatrix} \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \vdots \\ \Delta p_T \end{bmatrix}, \qquad (15)$$

where $\Delta P_i = P_t - P_{ref,t}$ and $\Delta p_t = p_t - p_{ref,t}$.

As shown in (15), the PEM treats the electricity of all time-periods as products that substitute or complement each other. Compared to the bids of traditional bidding mechanisms, (10) requires only one bid instead of 24 single bids and extends demand curve to the inter-temporal dimension. Compared Su and Kirschen's bidding mechanism [21], which investigates into specific load operation limits, the PEM models inter-temporal load shifting in a compact way. In addition, inter-temporal constraints either from demand side, such as end-users' shifting habits, or from system side, such as DR programs' types, can also be described by the PEM. Section 3.3 will describes in detail the attributes, forms and estimations of the PEM.

Sometimes a retailer (or load aggregator) need to construct one or multiple PEMs for a single or multiple groups of customers. The reason for not aggregating all the PEMs into one is to retain the each customer group's capacity information, $P_{max}(t)$ and $P_{min}(t)$. Thus, a retailer can classify the loads by end-user shifting types according to a unique PEM. The collection of these PEMs represents the overall end-user load shifting under this retailer. Section 3.4 will give more detail on this occasion.

3.2 Bidding Acceptance Rules and Settlement Rules

This section will discuss the bidding acceptance rules and price settlement under the proposed bidding mechanism in DA markets. The complementary price settlement rules in RT markets require PEM's transformation, and will be discussed in section 3.3. As stated in section 2.3, bidding acceptance rules and price settlement are separated rules and has different function: bidding acceptance rules define the objective function in a bidding process, while price settlement determines the bidders' final payment. However, in practice, these two sets of conditions are usually lumped together, because they are computed by solving the same problem. This is also true in DA market auctions under the proposed bidding mechanism.

This section formulates the two sets of conditions based on different DR programs. Section 3.2.1 gives the bidding acceptance rules and price settlement under traditional bidding mechanisms. The formulation in Section 3.2.1 is originally developed by Schweppe [11], and is extended in this thesis by adding inter-temporal generation constraints. Based on this basic formulation structure, Section 3.2.2 develops the two sets of conditions under the proposed bidding mechanisms. The obtained optimal price structures are compared to the one in Section 3.2.1. Section 3.3.3 further points out the market equilibrium under the proposed bidding mechanism. Moreover, it gives a brief overview on sensitivity analysis of power system contingencies under the proposed bidding mechanism.

3.2.1 Under Traditional Bidding Mechanism

In Section 2.3, we formulate the existing major bidding mechanisms in DA electricity markets as optimization problems. Bidding acceptance rules are formulated into the objective function of the optimization problem, and price settlement rules are specified by the problem's constraints. In this section, we will show how the bids are accepted and how the optimal price is settled by solving such an optimization problem. This mathematical demonstration is previously done by several works [4, 13, 25, 26] under different bidding acceptance rules and price settlement conditions. A most comprehensive one is the *spot pricing* structure developed by Schweppe [12]. However, this pricing structure is solved as a single-period "deterministic" model, which assumes inter-temporal independencies. After the *spot pricing* was developed, Motto [27] proposed a multi-period auction, which takes the generation inter-temporal constraints into account. Though a closed-form solution of the optimal price is developed in Arroyo's work, the pricing structure is not observable due to integer variables being involved. In this section, we present a general optimization problem for the traditional bidding mechanisms bases on Schweppes' work. Inter-temporal generation constraints are added to this formulation. In this thesis, we assume the formulation does not contain any binary variables, which indicate generation units' commitment states. Assuming bids are submitted according to the traditional bidding rules shown in (1), we formulate the problem as following:

Generation

There are J generation units on the supply side. Let $Y_j(t)$be output from unit j at hour t, a decision variable. Unit j has maximum output $P_{max}(t) = K_j$, minimum output $P_{min}(t) = 0$, and marginal generating cost $\frac{\partial C_j(Y_j(t))}{\partial Y_j(t)} = \lambda_j$. For convenience, units are numbered in order of operating costs, i.e. , $\lambda_1 \leq \lambda_2, \dots, \lambda_J$. Then the decision variable $Y_j(t)$ satisfy:

$$0 \leq Y_j(t) \leq K_j \; \forall j \in \mathcal{J} \quad (16)$$

In addition, unit j has multiple inter-temporal operation limits, which are described by P_{rpu} , P_{rpd} , t_{on} , t_{off} in (1). These limits are modeled by:

$$\underline{y_j} = < y_{j,1}, \dots, y_{j,m} > \quad \forall j \in \mathcal{J}, m \in \mathcal{M}_j \quad (17).$$

where m is the number of limits, and $\mathcal{M}_j$ is the set of inter-temporal limits of unit j. Each limit m depends on the operation status across the whole timeframe $\mathcal{T}$, and $y_{j,m} = y_{j,m}\left(Y_j(1), \dots, Y_j(T)\right)$. In practice, $y_{j,m}$ may only depend on several sequential hours of $\mathcal{T}$.

Demand

Since the market is wholesale competitive, the demand-side acting entities are retailers. Individual end users act independently. Their demands depend on time of day, weather, the price of electricity of current time, the price of other inputs, but independent of price and electricity use at other hours across the auction period. A critical step is that we shall model end users as price-taking expected profit-maximizing firms. Let F_i be the short-run value-added function end user i's use of electricity. Thus, it is i's profit, minus the cost of all non-electricity variable inputs. It depends on the end user's electricity use $D_i(t)$ and other random variables (which are ignored here for simplicity). Thus $F_i = F_i(D_i(t))$, and the end user will choose $D_i(t)$ to maximize its profit:

$$\text{max: profit for } i = F_i\big(D_i(t)\big) - p_i(t)D_i(t) \qquad (18)$$

$$\frac{\partial F_i\big(D_i^*(t)\big)}{\partial D_i^*(t)} = p_i(t) \qquad (19)$$

where $D_i^*(t)$ is the optimal use of electricity that maximizes its profit. Equation (19) implies $D_i(t) = D_i(p_i(t))$ because of end-user profit maximization. In other words, for price-taking expected profit-maximizing end users, their demand curve $D_i(p_i(t))$ gives their optimal electricity use under every given selling market price.

Transmission

A complete characterization of the network at hour t requires that we know the flows and losses along each line, and the net injections or withdrawals at each node. These are related by a number of equations, which are discussed in detail by Schweppe [11].

Let the flows along each line at hour t be given by the vector

$$\underline{Z}(t) = < Z_1(t), \dots, Z_K(t) >.$$

Then total real power losses throughout the network are:

$$l(t) = l\left(\underline{Z}(t)\right). \qquad (20)$$

An electric power system has an energy balance constraint:

$$\sum_j Y_j(t) = \sum_i D_i(t) + l(t). \qquad (21)$$

Other network constraints must also be observed. For simplicity, we only present transmission limits in this formulation. Additional constraints, such as voltage limits, can be added and consequently lead to similar final solutions under the same approach. In the formulation here, flows in each line must not exceed design limits, or the line will fail:

$$Z_{min,k} \leq Z_k(t) \leq Z_{max,k} \quad \forall k \in \mathcal{K}, \qquad (22)$$

where $Z_{min,k}$ and $Z_{max,k}$ are design limits for line k.

The power flows $\underline{Z}(t)$, in turn, depend on generation and demand at each node:

$$\underline{Z}(t) = \underline{Z}\left(\underline{Y}(t), \underline{D}(t)\right), \qquad (23)$$

where $\underline{Y}(t)$ and $\underline{D}(t)$ are the vectors of generation and demand augmented to have one element for each node.

Bidding acceptance rules

The bidding acceptance rules associated with the spot pricing structure is to maximize end-users' plus suppliers' surplus, which is the social welfare defined in Section 2.3, subject to the previously discussed constraints. These constraints depend on the suppliers' capital stock of generators and transmission lines. For pricing and operational decisions, we maximize short-run welfare with a fixed capital stock. This is, across the hours $t \in \mathcal{T}$, we wish to maximize, over generator output levels $Y_j(t)$ and end-user prices $p_i(t)$,

$$\text{max:} \sum_t \sum_i F_i(D_i(t)) - \sum_t \sum_j \lambda_j Y_j(t) \quad (24)$$

subject to constraints (16) and (17) for all generators $j \in \mathcal{J}$, inverse demand functions (19) for all end users $i \in \mathcal{I}$, and network constraints (20) to (23). Among these constraints, generation constraints are derived from the suppliers' bids, which are determined by the bidding rules, and network constraints, as established system conditions, are known in advance to the ISO. The demand constraints, however, are neither derived from retailers' bids nor a known condition. This is because traditional bidding mechanisms do not require regular demand-side participation, and the value-added function F_i is hard to estimate. In other words, the constraint (19) only exist here for the mathematical purpose and will not have a concrete form in practice.

Formulation solution

We now have a constrained optimization problem. Some of the dual variables will turn out to have interpretations as optimal prices; others will be the shadow values of additional capacity.

The Lagrangian to be maximized by the bidding host, who is usually the ISO, over all generation levels Y_j and over prices $p_i(t)$ is:

$$\mathcal{L}(t) = \underbrace{\sum_t \sum_i F_i(D_i(t))}_{\text{end-user value added}} - \underbrace{\sum_t \sum_j \lambda_j Y_j(t)}_{\text{generation cost}} - \underbrace{\sum_t \theta_t [\sum_j Y_j(t) - \sum_i D_i(t) - l(t)]}_{\text{energy balance constraint}}$$

$$- \underbrace{\sum_t \sum_j \mu_{j,t}[Y_j(t) - K_j]}_{\text{unit capacity constraint}} - \underbrace{\sum_j \sum_m \gamma_{j,m} y_{j,m}}_{\text{unit inter-temporal constraints}}$$

$$-\underbrace{\sum_t \sum_k (Z_k(t) - Z_{k,max})\eta_{k,max,t} + \sum_t \sum_k (Z_k(t) - Z_{k,min})\eta_{k,min,t}}_{\text{transmission line constraints, one pair per line}} \quad (25)$$

Here θ_t is the shadow price of another unit of demand at a "general" location. It will turn out to be the optimal spot price at one of the nodes which is arbitrarily chosen as the zero point for measurement. This node is termed the "swing bus" in power system parlance. The multiplier $\mu_{j,t}$ is the shadow value of extra generating capacity of type j at hour t. It is zero except when generator j at hour t is fully loaded.[1] $\gamma_{j,m}$ is the shadow value of the mth inter-temporal operation constraint of unit j. It is zero except when the operation constraint is bounded. $\eta_{k,t}$ is the shadow value of additional transmission capacity:

$$\eta_{k,t} = \eta_{k,max,t} - \eta_{k,min,t} \qquad (26)$$

It is nonzero when line k is fully loaded. It is positive if the line is fully loaded in the "forward" direction; negative if the line is fully loaded in the "backwards" direction.

Price settlement

The Lagrangian (25) can be differentiated with decision variables $D_i(t)$ and $Y_j(t)$ to obtain the first-order conditions for the various generator outputs Y_j and end-user consumption D_i. By substituting the demand constraint (19) in, we get the price p_i^* that optimizes the short-run welfare function (24) and maximizes end-user profit (18). In other words, by setting prices at $p_i^*(t)$, the market will give a demand $D_i^*(t)$, since end users are expected profit-maximizing, that also optimizes short-run social welfare due to $D_i^*(t)$ satisfies the first-order condition of the Lagrangain $\mathcal{L}(t)$. The optimal price at hour t for end user i turns out to be:

$$p_i^*(t) = \theta_t \left[1 + \frac{\partial l(t)}{\partial D_i(t)}\right] + \sum_k \frac{\partial Z_k(t)}{\partial D_i(t)} \eta_{k,t} \qquad (27)$$

[1] Since the formulation can be proved as non-degenerate, its optimal solutions satisfies strict slackness complementary conditions.

The structure of this optimal price is:

optimal spot price to i = [social cost of additional demand at the swing bus]

× [1+ incremental losses caused by *i*]

+ [transmission constraint terms, summed over lines].

This equation settles the price of electricity at time t and location i. This price, since derived from the prospect of end-user profit maximization, is called end-user price or market selling price[2]. Previous works states that generation price or market buying price can be derived by considering generation as negative demand. Section 3.2.2 will show that this statement is not rigorous and only holds for some bidding mechanisms.

In (27), θ_t is the same for all end users. Define system lambda $\lambda(t)$ as the short-run marginal generating cost. It is the cost of generating another MWh of electricity from a marginal unit, then getting it back to the swing bus despite losses and transmission constraints.

Then

$$\theta_t = \lambda(t) + \mu(t) + \sum_m \gamma_m(t) \frac{\partial y_{j,m}}{\partial Y_j(t)}, \qquad (28)$$

of which the structure is:

[shadow price] = [marginal generating cost] + [curtailment premium]

+ [smoothing operation premiums].

Here $\mu(t)$ is the premium needed to curtail demand back to available supply, when rationing would otherwise be necessary. The last term, $\sum_m \gamma_m(t) \frac{\partial y_{j,m}}{\partial Y_j(t)}$, is the premiums for loads' smoothing operation. This term penalizes the inter-temporal demands' change that bounds the generating operation limits.

[2] the "selling" and "buying" are defined from the ISO's prospective, i.e. if the ISO sells electricity at a certain price, then the price is called the selling price, vice versa.

To show how θ_t can be decomposed, consider the case where there is at least one marginal generating unit which operates within its inter-temporal limits, i.e. there exists a unit M satisfying:

$$0 < Y_M(t) < K_M$$

and

$$y_{M,m} < 0 \quad \forall m \in \mathcal{M}_M.$$

Then by strict complementary slackness, the shadow value of capacity, $\mu_{M,t}$, and the shadow value of generating inter-temporal limits, $\gamma_{M,m,t}$, are temporarily zero. Therefore, $\theta_t = \lambda(t)$, and substituting this result in equation (27) the system marginal cost becomes:

$$\lambda(t) = \frac{\lambda_M + \sum_k [\frac{\partial Z_k(t)}{\partial Y_M(t)} \eta_{k,t}]}{1 - \frac{\partial l(t)}{\partial Y_M(t)}}. \qquad (29)$$

Here λ_M is the direct cost of incremental generation at M.

Consider now the other case in which all variable units are fully used, but none of them touches the inter-temporal generating limits. Then all available units j have local price higher than cost:

$$p_j^*(t) = \lambda_j + \mu_{j,t} > \lambda_j. \qquad (30a)$$

We can still define $\lambda(t)$ by using equation (29) for the most expensive unit, J, substituted for M in the equation. However, because of (30a) we need a positive curtailment premium, $\mu(t) = \mu_{J,t} > 0$, to make the equations balance.

Consider another case in which a marginal unit can be found, that is, there is sufficient generation capacity for demand. However, this marginal unit touches some of its inter-temporal limits, $y_{M,m}$. Assume there is only one inter-temporal generating limit for every generation unit, and let this limit be the ramp-up constraint. Then the marginal unit has a local price:

$$p_M^*(t) = \lambda_M + \gamma_{M,rpu} > \lambda_M. \qquad (30b)$$

The equation (28) has $\mu(t) = 0$ and $\gamma(t) = \gamma_{M,rpu} > 0$.

Thus, when there is a generator on the margin, $\mu(t)$ is zero. It becomes nonzero whenever rationing would otherwise be needed to avoid excess demand. Moreover, $\gamma(t)$ sets the premium of the marginal load's inter-temporal operation based on the marginal generation unit's inter-temporal limits. In the example of ramp-up constraints, $\gamma_{M,rpu}$ rations a sudden demand boost at hour t.

3.2.2 Under Proposed Bidding Mechanism

A critical part of spot pricing structure is the "social cost of additional demand at swing bus", θ_t, which is defined by the marginal unit M (or the last available unit J, if demand exceeds the generation capacity) of a power system. In traditional bidding mechanisms, the end-user added-value functions $F_i(D_i)$ is not required in the bidding rules, and therefore the ISO cannot calculate the social optimal $D_i(t)$. For this reason, the marginal unit M is defined under a predicted total demand $D_\Sigma(t)$ for the next 24 hours.

This definition cause two major problems: (1) the obtained optimal price does not maximize welfare as designed in the bidding acceptance rule, but only minimizes the generation cost; (2) since the total demand $D_\Sigma(t)$ is neither an elastic quantity in the bidding mechanisms nor predicted at the market equilibrium, it is possible to generate a unstable optimum, which lead to a total demand different from $D_\Sigma(t)$ under any Demand Response (DR) program. In other words, lack of end-user value-added functions or equivalent information, by the established formulation it is hard to find a price that allows end users to maximize their profit while optimizing the social welfare at the same time.

For this reason, the proposed bidding mechanism defines the bidding rules shown in equation (10). Section 3.1 has shown that from the proposed bidding rules, we can derive the inter-temporal demand curves of end users. With these bidding rules, the bidding mechanism defines two sets of bidding acceptance rules and price settlements according to the two primary DR categories: time-based rates and incentive-based demand response.

Time-based rates

Assume end users are price taking expected profit-maximizing firms. In addition, assume that all generation condition described by suppliers' bids and transmission condition known by the ISO are the same as in Section 3.2.1. Compared to end-user profit defined by equation (18), end user i under time-based rates chooses their electricity use $D_i(t)$ to maximize its profit over the rates' timeframes. Assume the rates' timeframes are consistent with the auction period, $\mathcal{T}$. The end user i's profit function is subject to its consumption range and inter-temporal constraints $d_{e,n} = d_{e,n}\big(D_i(1), \dots, D_i(T)\big) \le 0$:

$$\text{max: profit for } i = \sum_t F_i(D_i(t)) - \sum_t p_i(t) D_i(t) \quad (31)$$

$$s.t. \ \underline{d_e} = < d_{e,1}, \dots, d_{e,n} >, \quad (32)$$

$$D_{i,min} \le D_i(t) \le D_{i,max}, \quad \forall t \in \mathcal{T} \ (33)$$

where $\underline{d_e}$ is the vector of the inter-temporal constraints of an end-user group $e \in \mathcal{E}$. The demand curve that maximizes the end user i's profit is:

$$\frac{\partial F_i\big(D_i^*(t)\big)}{\partial D_i^*(t)} + \sum_n \frac{\partial d_{e,n}}{\partial D_i^*(t)} v_{e,n} + \rho_{i,t} = p_i(t), \quad (34)$$

where $v_{n,t}$ is the shadow value of end user i's inter-temporal constraint $d_{e,n}$. $\rho_{i,t}$ is the shadow value of a larger electricity consumption range:

$$\rho_{i,t} = \rho_{i,max,t} - \rho_{i,min,t}, \quad (35)$$

It is nonzero when demand $D_i(t)$ is within the bid consumption range. It is positive if the demand to maximize the profit gets the least or most consumption value. Equation (34) is the general form of equation (19). This demand-side information is either absent or incomplete in traditional bidding mechanisms.

Bidding acceptance rules

The bidding acceptance rules under time-based rates programs are to maximize social welfare as equation (24):

$$\max: \sum_t \sum_i F_i(D_i(t)) - \sum_t \sum_j \lambda_j Y_j(t) \quad (24)$$

subject to constraints (16) and (17) for all generators $j \in \mathcal{J}$, inverse demand functions (34) and constraints (33) for all end users $i \in \mathcal{I}$, the inter-temporal constraints of all end-user groups $e \in \mathcal{E}$, and network constraints (20) to (23). We now have a constrained optimization problem. The Lagragian to be maximized over all generation levels Y_j and over prices $p_i(t)$ is:

$$\mathcal{L}(t) = \underbrace{\sum_t \sum_i F_i(D_i(t))}_{\text{end-user value added}} - \underbrace{\sum_t \sum_j \lambda_j Y_j(t)}_{\text{generation cost}} - \underbrace{\sum_t \theta_t [\sum_j Y_j(t) - \sum_i D_i(t) - l(t)]}_{\text{energy balance constraint}}$$

$$- \underbrace{\sum_t \sum_j \mu_{j,t}[Y_j(t) - K_j]}_{\text{unit capacity constraints}} - \underbrace{\sum_j \sum_m \gamma_{j,m} y_{j,m}}_{\text{unit inter-temporal constraints}}$$

$$- \underbrace{\sum_t \sum_i [(D_i(t) - D_{i,max})\rho_{i,max,t} + (D_i(t) - D_{i,min})\rho_{i,min,t}]}_{\text{demand range constraints, one pair per end user}} - \underbrace{\sum_e \sum_n d_{e,n} v_{e,n}}_{\text{end-user inter-temporal constraints}}$$

$$- \underbrace{\sum_t \sum_k [(Z_k(t) - Z_{k,max})\eta_{k,max,t} + (Z_k(t) - Z_{k,min})\eta_{k,min,t}]}_{\text{transmission line constraints, one pair per line}} \quad (35)$$

Price settlement

The Lagrangian (35) can be differentiated to obtain the first-order conditions for the various generator outputs Y_j and end-user electricity use D_i. By substituting the demand constraint (34) in, we get the price p_i^* that optimizes the short-run welfare function (24) and maximizes end-user profit (31). The optimal price at hour t for end user i turns out has the spot pricing structure shown in equation (27):

$$p_i^*(t) = \theta_t \left[1 + \frac{\partial l(t)}{\partial D_i(t)}\right] + \sum_k \frac{\partial Z_k(t)}{\partial D_i(t)} \eta_{k,t} \qquad (27)$$

Equation (27) concludes that time-based rates d not change the optimal spot pricing structure. This is because the end-user constraints (32) and (33) appear in both the end-user profit maximization formulation (31) and the social welfare formulation (24) problem. These two formulations imply that end users schedule their electricity use with the knowledge of DA prices and the ISO sets the DA price with end-user consumption patterns. Therefore, the optimal pricing structure will not be affected by specific end-user consumption patterns e. This conclusion disclaims David [13] optimal pricing structures, which are different for the short-term, long-term and real-world customers. Under David's proposed pricing structure, the ISO sets the price regardless end-user consumption limits. Therefore, the obtained price will lead to an infeasible generation and demand output.

To calculated equation (27), we transform the demand curve (34) into its PEM form in (15):

$$\underline{D_i} - \underline{e} D_{i,ref} = \varepsilon_{T\times T}\left(\underline{p_i} - \underline{e} p_{i,ref}\right), \quad (15b)$$

where $\underline{e}$ is a unit vector of dimension T, and $\underline{D_i}$, $\underline{p_i}$ are vector of demand and prices at hours $t \in \mathcal{T}$.

If the PEM $\varepsilon_{T\times T}$ is non-singular[3], we define an inverse matrix $\xi_{T\times T}$:

$$\xi_{T\times T} = \varepsilon_{T\times T}^{-1}, \qquad (36)$$

and

$$\xi_{t\tau} = \frac{\partial p_i(t)}{\partial D_i(\tau)}, \qquad (37)$$

[3] if the PEM is singular, a fine adjustment can make it inversible.

where $\xi_{t\tau}$ is the row t and column τ entry of $\xi_{T\times T}$.

Since the PEM is known from the retailer i's bids, we can derive $\xi_{t\tau}$, and substitute equation (27) into equation (37), which sets the accepted bids Y_j and market price p_i. The optimum set by this approach, since calculated under elastic demand, is the market equilibrium, which is stable under DR programs and optimizes social welfare.

Incentive-based demand response

An incentive-based DR program can be either an emergency or a regular DR program. The entities such as the ISO who initiate these programs, when needed, will call their customers. Instructions are included in these calls, telling the customers how much electricity use to change. The customers are paid by contracted incentives if they respond to the call, and are penalized if they do not respond. The incentives and penalties are predefined in the DR programs' contracts in advance. The customers do not know when they will be called in advance. Assume there are F incentive-based DR programs in our DA market. $\mathcal{F}$ is the complete set of these DR programs. Incentive-based DR program $f \in \mathcal{F}$ defines its instruction at hour t is $a_f(t)$. Let the time set $\mathcal{T}_f$ be the hours at which DR program f will call its customers, then the instruction vector for all the DR programs on the market is:

$$\underline{a} =< a_1(t), \ldots, a_F(t) >, \quad (38)$$

where

$$a_f(t) = a_f\big(D_1(t), \ldots, D_i(t)\big) \leq 0, \;\; \forall t \in \mathcal{T}_f, i \in f \cap \mathcal{I} \; and \; f \in \mathcal{F} \quad (39)$$

Since end users who contract with the DR programs do not know when they will be called in advance, the end users maximize their profit under the same formulation as equations (31) to (33). This formulation in turn leads to the demand curve of equation (34).

Bidding acceptance rules

The DR programs' constraints (39) are known by the ISO, who conducts the DA auction. Therefore, the bidding rules are to maximize end users' plus suppliers' surplus:

$$\text{max:} \sum_t \sum_i F_i(D_i(t)) - \sum_t \sum_j \lambda_j Y_j(t), \quad (40)$$

subject to constraints (16) and (17) for all generators $j \in \mathcal{J}$, inverse demand functions (34) and constraints (33) for all end users $i \in \mathcal{I}$, the inter-temporal constraints of all end-user groups $e \in \mathcal{E}$, network constraints (20) to (23), and constraints describing all incentive-based DR programs $f \in \mathcal{F}$ (38).

We now have a constrained optimization problem. The Lagragian to be maximized over all generation levels Y_j and over prices $p_i(t)$ is:

$$\mathcal{L}(t) = \quad (41)$$

$$\underbrace{\sum_t \sum_i F_i(D_i(t))}_{\text{end-user value added}} - \underbrace{\sum_t \sum_j \lambda_j Y_j(t)}_{\text{generation cost}}$$

$$\underbrace{+\sum_t \theta_t [\sum_j Y_j(t) - \sum_i D_i(t) - l(t)]}_{\text{energy balance constraint}} - \underbrace{\sum_t \sum_j \mu_{j,t}[Y_j(t) - K_j]}_{\text{unit capacity constraints}} - \underbrace{\sum_j \sum_m \gamma_{j,m} y_{j,m}}_{\text{unit inter-temporal constraints}}$$

$$\underbrace{-\sum_t \sum_i [(D_i(t) - D_{i,max})\rho_{i,max,t} + (D_i(t) - D_{i,min})\rho_{i,min,t}]}_{\text{demand range constraints, one pair per end user}} - \underbrace{\sum_e \sum_n d_{e,n} v_{e,n}}_{\text{end-user inter-temporal constraints}}$$

$$\underbrace{-\sum_t \sum_k [(Z_k(t) - Z_{k,max})\eta_{k,max,t} + (Z_k(t) - Z_{k,min})\eta_{k,min,t}]}_{\text{transmission line constraints, one pair per line}} - \underbrace{\sum_f \sum_{t \in \mathcal{T}_f} a_f(t)\omega_{f,t}}_{\text{incentive-based DR constraints}}$$

Price settlement

The Lagrangian (41) can be differentiated to obtain the first-order conditions for the various generator output Y_j and the retailer electricity consumption D_i. Assume By substitute the demand constraint (34) in, we get the price p_i^* that optimizes the short-run welfare function (40) and maximizes end-user profit (31). The optimal price at hour t for retailer i turns out to be:

$$p_i^*(t) = \theta_t \left[1 + \frac{\partial l(t)}{\partial D_i(t)}\right] + \sum_k \frac{\partial Z_k(t)}{\partial D_i(t)} \eta_{k,t} + \sum_f \frac{\partial a_f(t)}{\partial D_i(t)} \omega_{f,t}, \quad (42)$$

which has a structure as:

optimal price to i under f = [social cost of additional demand at the swing bus]
× [1+ incremental losses caused by i]
+ [transmission constraint terms, summed over lines]
+ [DR constraint terms, summed over DR programs]

From this equation (42), we have the following results about the value of energy at hour t and location i under DR programs f:

Result 1

The DR constraint terms of the optimal price provide additional incentives, which motivate end users fully response. Assume the DR instruction f at hour t and location i is to reduce demand, then $\frac{\partial a_f(t)}{\partial D_i(t)} \geq 0$ according to equation (39). The DR constraint's shadow value $\omega_{f,t}$ is positive when end user i does not fully respond and only satisfies marginal requirements, $a_f(t) = 0$. In this case, the DR constraint term $\frac{\partial a_f(t)}{\partial D_i(t)} \omega_{f,t}$ becomes positive, and the price $p_i^*(t)$ increases. This higher price will discourage i's electric use, and thus acts as an additional incentive (or penalty) to the DR program. On the other hand, when end user i fully responds, shadow value $\omega_{f,t}$ is zero. Thus, the DR constraint term $\frac{\partial a_f(t)}{\partial D_i(t)} \omega_{f,t}$ will disappear the price is lower.

Result 2

In a power system, two points geographically different at $i1$ and $i2$, contracted with different incentive-based DR programs become separate markets for electricity, with different electricity rates. Assume no losses and transmission limits exit, then the price difference between the two points become:

$$p_{i2}^{*}(t) - p_{i1}^{*}(t) = \sum_{f \in \mathcal{F}(i1)} \frac{\partial a_f(t)}{\partial D_i(t)} \omega_{f,t} - \sum_{f \in \mathcal{F}(i2)} \frac{\partial a_f(t)}{\partial D_i(t)} \omega_{f,t}, (43)$$

where $\mathcal{F}(i1)$ and $\mathcal{F}(i2)$ are DR programs contracted at location $i1$ and $i2$.

Result 3

The optimal price induce the demand that end users will chose to maximize their profits. Moreover, the DR programs to call and electricity to adjust can be calculated under this price. By substituting equation (42) into equation (37), the optimal demand is as $D_i^*(t)$, and the optimal curtailment is $D_{io}(t) - D_i^*(t)$. If adding a binary decision variable $q_f(t)$, which equals to 1 when DR program f is called and equals to 0 otherwise, to the optimization formulation, we can find the optimal DR schedule by calculating $q_f(t)$.

The proposed v. s. traditional bidding mechanisms

In traditional bidding mechanisms, the demand is predicted as an elastic quantity. Our proposed bidding rules requires demand responsive information in price elasticity matrices. It is misleading, however, to think that the proposed bidding mechanism dispatches loads. Not considering Demand Side Bid in reserve market, regular demands are un-dispatchable. The purpose of accepting bids from retailers is to require demand-side information. Based on the information, the proposed bidding mechanism sets a price that induces socially optimal behavior, and dispatches the generation units.

The differences between these two sets of bidding rules lead to distinctive approaches of their bidding acceptance rules and price settlement: the traditional bidding mechanisms determine the optimal price under the predicted demand; while the proposed bidding mechanism sets the price that can induce socially optimal behavior, relying on end-user profit maximization. For this reason, the proposed bidding mechanism is more stable and remains its optimality even with demand-side participation.

3.2.3 Sensitivity Analysis of the Proposed Bidding Mechanism

This section conducts the local sensitivity analysis of the proposed bidding mechanism. Under incentive-based DR programs, the optimization formulation of the proposed bidding mechanism is:

$$\max: \sum_t \sum_i F_i(D_i(t)) - \sum_t \sum_j \lambda_j Y_j(t), \quad (40)$$

s .t.

Generation constraints:

$$0 \le Y_j(t) \le K_j, \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (16)$$

$$y_{j,m} = y_{j,m}\left(Y_j(1), \dots, Y_j(T)\right) \le 0, \quad \forall j \in \mathcal{J}, m \in \mathcal{M}_j \quad (17b)$$

Demand constraints:

$$\frac{\partial F_i(D_i(t))}{\partial D_i(t)} + \sum_n \frac{\partial d_{e,n}}{\partial D_i(t)} v_{e,n} + \rho_{i,t} = p_i(t), \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \quad (34)$$

$$s.t.\ \underline{d_e} = < d_{e,1}, \dots, d_{e,n} >, \qquad \forall e \in \mathcal{E}, n \in \mathcal{N}_e \quad (32)$$

$$D_{i,min} \le D_i(t) \le D_{i,max}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \ (33)$$

$$\underline{D_i} - \underline{e} D_{i,ref} = \varepsilon_{T\times T}\left(\underline{p_i} - \underline{e} p_{i,ref}\right), \qquad (15b)$$

$$\xi_{T\times T} = \varepsilon_{T\times T}^{-1}, \qquad (36)$$

$$\xi_{t\tau} = \frac{\partial p_i(t)}{\partial D_i(\tau)}, \qquad \forall\, t, \tau \in \mathcal{T} \ (37)$$

Transmission network constraints:

$$l(t) = l\left(\underline{Z}(t)\right), \qquad \forall\, t \in \mathcal{T}(20)$$

$$\underline{Z}(t) = < Z_1(t), \dots, Z_K(t) >, \qquad \forall t \in \mathcal{T}(20b)$$

$$\underline{Z}(t) = \underline{Z}\left(\underline{Y}(t), \underline{D}(t)\right), \qquad \forall t \in \mathcal{T} \ (23)$$

$$\underline{Y}(t) = < Y_1(t), \dots, Y_J(t) >, \qquad \forall t \in \mathcal{T} \ (23b)$$

$$\underline{D}(t) = < D_1(t), \dots, D_I(t) >, \qquad \forall t \in \mathcal{T} \ (23c)$$

$$\sum_j Y_j(t) = \sum_i D_i(t) + l(t), \quad \forall t \in \mathcal{T} \ (21)$$

$$Z_{min,k} \leq Z_k(t) \leq Z_{max,k}, \quad \forall k \in \mathcal{K}, t \in \mathcal{T} \quad (22)$$

Incentive-based DR constraints:

$$\underline{a} =< a_1(t), \dots, a_F(t) >, \quad \forall t \in \mathcal{T}, f \in \mathcal{F} \ (38)$$

$$a_f(t) = a_f\big(D_1(t), \dots, D_i(t)\big) \leq 0, \ \ \forall t \in \mathcal{T}_f, i \in (f \cap \mathcal{I})(39)$$

Assume the above formulation can be linearized into the linear programming standard form as:

$$z = \min\ c'x, \qquad (44)$$

$$s.t. Ax = b,$$

$$x \geq 0\,,$$

where x is the vector of all decision variables: $p_i(t)$ and $Y_j(t)$, z is the bidding acceptance rule and (A, b) is the constraint matrix.

Assume problem (40) is feasible, then its linearized standard form (44) has a basic feasible solution x. Let $B(1), \dots, B(T)$ be the indices of the basic variables, and let $B = [A_{B(1)}, \dots, A_{B(T)}]$ be the corresponding basis matrix. In particular, we have $x_i = 0$ for every nonbasic variable, while the vector $x_B =< x_{B(1)}, \dots, x_{B(T)} >$ of basic variables is given by:

$$x_B = B^{-1}b.$$

Let c_B be the vector of costs of the basic variables. For each $\bar{\imath} \in \mathcal{I} \cup \mathcal{J}$, we define the reduced cost $\bar{c}_{\bar{\jmath}}$ of the variable $x_{\bar{\jmath}}$ according to the formula:

$$\bar{c}_{\bar{\jmath}} = c_{\bar{\jmath}} - c_B' B^{-1} A_{\bar{\jmath}}.$$

The basis B is optimal when it satisfies:

1. Feasibility conditions: $B^{-1}b \geq 0$,
2. Optimality conditions: $c' - c_B' B^{-1} A \geq 0'$.

Changes in c

Disturbances of generation units' cost causes changes in c. By checking the optimal basis conditions, we find the feasibility conditions are unaffected and the optimality conditions are affected. In addition, problem (40) is formulated after unit commitment, thus all generation decision variables x_j $(j \in \mathcal{J})$ are associated with committed units. In other words, $x_j (j \in \mathcal{J})$ are strict positive, and thus are basic variables. Therefore, if the generation unit j's cost is disturbed by Δ, the current basis remains optimal when Δ satisfies:

$$\max_{\bar{a}_{tj}<0}: \frac{\bar{c}_j}{\bar{a}_{tj}} \leq \Delta \leq \min_{\bar{a}_{tj}>0}: \frac{\bar{c}_j}{\bar{a}_{tj}}\ , \quad (45)$$

where $\bar{a}_{tj} = [B^{-1}A]_{tj}$. In this case, the change of social welfare in equation (44) is $\sum_j \lambda_j \Delta$.

When Δ reaches out of the range in problem (40), the new optimal basis can be found by applying primal simplex method.

Changes in b

Disturbances in transmission limits, sudden losses of generation capacities or transmission lines, and disturbances in the PEM can be modeled by changes in b. By checking the optimal basis conditions, we find the optimality conditions are unaffected but the feasibility conditions are affected. Therefore, assume b changes by Δ, the current basis remains the optimal basis if:

$$\max_{\beta_{t\bar{j}}<0}: \left(-\frac{\bar{b}_t}{\beta_{t\bar{j}}}\right) \leq \Delta \leq \min_{\beta_{t\bar{j}}>0}: \left(-\frac{\bar{b}_t}{\beta_{t\bar{j}}}\right), \quad (46)$$

where $\beta_{t\bar{j}} = [B^{-1}]_{t\bar{j}}$, and $\bar{b}_t = [B^{-1}b]_t$. Consequently, the change of social welfare in problem (40) is $\sum_t b_t \Delta$.

When disturbance Δ gets out of this range, the current solution is no longer feasible but satisfies the optimality conditions. In this case, we can find the new optimal solution by applying dual simplex method.

Disturbances in A

Changes in instructions of incentive-based DR programs can be modeled by changes in A such as: a new variable is added and a new constraint is added .

If the disturbance accords to a new variable added to A. By checking the optimal basis conditions, we know that the feasibility conditions are unaffected. If the $t+1$th new variable satisfies:

$$c_{t+1} - c_B' B^{-1} A_{t+1} \geq 0, \qquad (47)$$

the current basis remains optimal. Otherwise, we apply primal simplex method to find the new optimal solution.

In the other case, if the disturbance accords to a new constraint added to A, the optimality conditions are unaffected. If the current solution satisfies the added constraint, then the current solution is feasible as well as optimal. Otherwise, we apply dual simplex to find the new optimal solution.

3.3 The Price Elasticity Matrix

Section 3.1 and Section 3.2 briefly introduce the Price Elasticity Matrix (PEM)'s role in the proposed demand responsive bidding mechanism. A PEM is a matrix $\varepsilon_{T\times T}$ consist of price-to-electric-use elasticities in the concerned timeframe T (see equation (15)):

$$\Delta P = \varepsilon_{T\times T}\Delta p$$

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_T \end{bmatrix} = \begin{bmatrix} \epsilon_{11} & \epsilon_{11} & \cdots & \epsilon_{1T} \\ \epsilon_{21} & \epsilon_{22} & \cdots & \epsilon_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon_{T1} & \epsilon_{T2} & \cdots & \epsilon_{TT} \end{bmatrix} \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \vdots \\ \Delta p_T \end{bmatrix}, \quad (15)$$

The PEM treats the electric use of all hours in T as products that substitute or complement each other. Therefore, together with the reference points required in the proposed bidding rules, the PEM constructs an inter-temporal dimensioned demand curve, which provides demand response information.

This section further investigates into the PEM's characteristics and establishment. Section 3.3.1 presents the PEM's classifications. Section 3.3.2 focuses on the factors that affect the PEM's establishment, and the PEM's transformations in the proposed bidding mechanisms. Section 3.3.3 gives a brief introduction on the PEM's estimation.

3.3.1 Classifications of the Price Elasticity Matrix

Under given DR programs, the ability of an end user to respond is influenced by both technical and economic factors. The load of a single or multiple end users is disaggregated into four load types: fixed, curtailable, distributed generation, and shiftable. The shiftable load type covers the on-site storage. To represent these four load types, the PEM is used. This representation is discussed in several previous works [6, 13, 25, 28, 29], but it has not yet been applied in a bidding mechanism.

Fixed loads are inelastic to price, and therefore all entries for this load type are equal to zero in the PEM. Curtailable loads represent *inessential* loads that can be shed (but not shifted) in the presence of high prices or incentives. Distributed generation is similar to a curtailable load in the sense that it is turned on during high prices and essentially contributes to a negative load. Both the curtailable loads and local generation are represented by a PEM with negative values along the diagonal and zero values for all off-diagonal entries. Shiftable loads can be moved to other periods during the day, but the total amount must be preserved under the *lossless* assumption. Load shifting is said to be lossless if the electric use amount is the same as before it is shifted. In the PEM, this characteristic is described as:

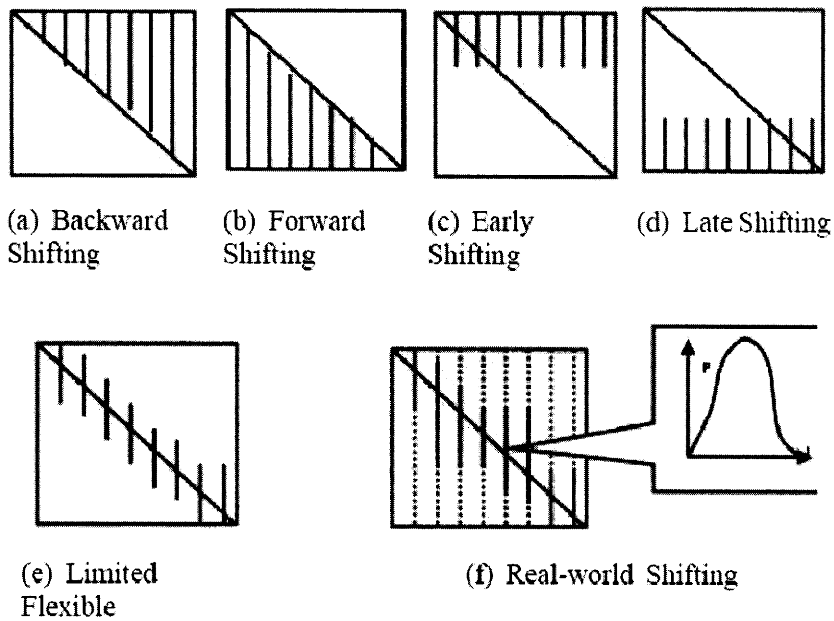
$$\sum_{t} \epsilon_{t\tau} = 0. \quad (48)$$

where $\epsilon_{t\tau}$ is the row t and column τ entry of the PEM matrix $\varepsilon_{T\times T}$.

For this last type, the PEM has negative valued on-diagonal entries and positive valued off-diagonal entries which satisfy equation (48).

For distributed generation and on-site storage, they will only operate when their marginal costs are below the market prices or incentives. Moreover, the shiftability of distributed generation and on-site storage is constrained by their capacity and operation limits.

To characterize end-user shifting behaviors, we consider the topologies of the PEM. Recall the PEM's definition in Section 3.1. For a PEM, its diagonal elements are the self-elasticities and the off-diagonal elements are cross-elasticities. Column τ of this matrix indicates how a change in price during the single hour $j$affects the demand during all the other hours across the concerned timeframe T. Fig. 3.3.1 illustrates the structure of the elasticity matrices corresponding to these various types of consumer reactions.



(a) Backward Shifting (b) Forward Shifting (c) Early Shifting (d) Late Shifting

(e) Limited Flexible (f) Real-world Shifting

Fig. 3.3.1 End-user response types modeled by PEMs.

If the only nonzero elements in this column are above the diagonal, the consumers react to a high price by moving their consumption earlier (*Backward Shifting*). If they are below the diagonal, they postpone their consumption until after the high price period (*Forward Shifting*). If consumers have the ability to reschedule their production over a long period, the nonzero elements will be spread widely over the column, probably with more preference of shifting around their original schedules (*Real-World Shifting*). On the other hand, if their flexibility is limited, the nonzero elements will be clustered around the diagonal (*Limited Flexibility Shifting*). Further classification under *Limited Flexibility Shifting* can be obtained by examining the specific shifting flexibility of end users [30]. Some customers may also decide that, if they have to reschedule their electricity consumption, they might as well take advantage of the hours of lowest prices, which typically are in the early hours of the morning (*Early Shifting* and *Late Shifting*).

3.3.2 Factors Affecting the PEM's Establishment

Several factors affect the PEM's establishment from the demand side or the system side. Demand-side factors include by end-user shifting behaviors and loads' physical characteristics, which are invisible from a central perspective such as the ISO. For this reason, we address in this section the system-side factors in terms of the two types of DR programs. Knowing these factors will help us better design and implement DR programs. We will examine the demand-side factors in the next section.

Time-based rates

Time-based rates affect the PEM's establishment by its designed timeframe, which determines the non-trivial entries of the PEM. As defined in Section 3.1, the dimension of the PEM, T, equals the bidding mechanism's transaction period, which further depends on the weather forecasting ability or specific market environments . Then for a T by T PEM, $\varepsilon_{T\times T}$, its effective-price hours and influenced-demand hours are defined by its non-trivial rows and columns. Collect all non-trivial rows T_a and non-trivial columns T_b of $\varepsilon_{T\times T}$ into a new matrix of dimension $|T_a|$ by $|T_b|$. Mathematically, this $\varepsilon_{|T_a|\times|T_b|}$ can be interpreted as a map from a $|T_a|$-dimensional price space to a $|T_b|$-dimensional demand space, illustrated in figure 3.3.2.

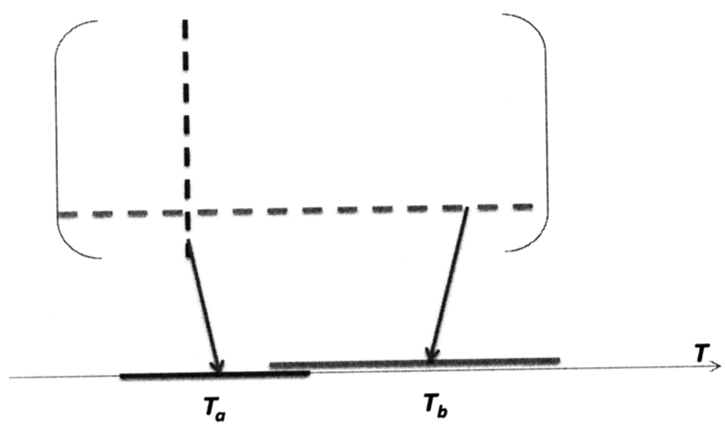


Fig. 3.3.2 The row dimension and column dimension of a PEM reflects affecting and affected periods in the timeframe of interest.

For $\varepsilon_{|T_a|\times|T_b|}$, its rows reflect the hours at which the prices take effect, and its column T_a reflect the hour at which the demand can be influenced. Viewed on the same time axis, hours in T_a and T_b can be the same, overlapping or departure from each other.

For this reason, in $\varepsilon_{|T_a|\times|T_b|}$ the self-elasticities are not necessarily on diagonal any more. The hours in T_b are limited to a proper sub-set of the hours approachable from T_b. The hours in T_a is determined by the internal attributes of the consumers' group. In other words, the PEM relates changes in demand to changes in prices within a single scheduling period. Changes in demand due to unusual prices in a previous period must be carried over separately in the load forecast. Therefore, an effective DR program design will make full use of the hours in T_b.

On the other hand, the PEM's entries of hours in T_a indicates end users' total energy change after scheduling. Remember that we define *lossless* by equation (48). Likewise, if end users' total energy change after scheduling is reduced, then: $\sum_t \epsilon_{t\tau} < 0$; otherwise: $\sum_t \epsilon_{t\tau} > 0$. Therefore, energy-saving DR programs should set their implementing time during a period in T_a, such that

$$\sum_t \epsilon_{t\tau} < \alpha, \qquad (49)$$

where α describes the magnitude of which the total energy can be saved after rescheduling.

The consumers' ability to react to unusual electricity prices varies with the time of day. One column of the elasticity matrix can therefore not be deduced from another through a simple translation along the diagonal. By constructing a price elasticities database depicted in figure 3.3.3, we can establish a PEM under a time-based rate program during any hours. By selecting DR programs' starting timings and their timeframes, the DR programs utilize different parts of the database. This database can be repeated to cover all possible starting timings and timeframes. For example, if T_b is from 0:00 to 23:00 and T_a is from the six hours before the scheduling hour to the six hours after the scheduling hour. The DR program's starting timing is 21:00 and its timeframe is 24 hours. To construct the PEM under this DR program, we replicate the existing database and connect the two databases at 23:00 and 0:00. Thus the new database becomes a $|T_a|\times 2|T_b|$ matrix. The PEM is obtained by selecting the entries of the corresponding hours in the database and setting all rest entries as zeros.

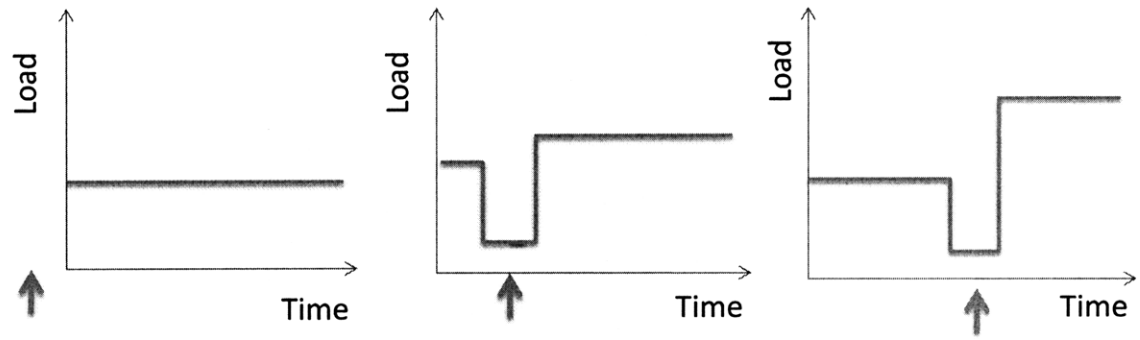


Fig. 3.3.4 Notification timing of incentive-based demand response programs on load profiles.

Transformations of the PEMs

The proposed bidding mechanism can be applied to more occasions with the PEMs' transformations. These occasions include: in RT markets, for industrial end users, under production management and for end user sensitive to other factors.

In RT Markets:

In RT markets, no bids are submitted or accepted for lack of transition time. The generation units and demand directly respond to the RT price released. In traditional bidding mechanism, the ISO calculated the RT prices based on generation bids of DA market, updated demand forecasts and real-times conditions in ancillary markets. In the proposed bidding mechanism, however, since the DA prices are calculated considering demand response and thus at the market equilibrium, there is no need to update demand forecasts in real time. Nevertheless, the demand curve may change due to exogenous factors such as weather. For this reason, fine adjustment of PEMs' entries and of reference points are needed to compute the optimal RT prices.

Moreover, end users cannot reschedule the electric use of the hours before the RT is released. Therefore, assume the RT price is calculated at hour τ, then the ISO should update the PEM by setting its columns before τ to zero. The final payments of the DA and RT markets are settled by either the two-settlement or the post-settlement system.

For industrial end users

A transformation of the PEM is to Demand Redistribution Matrix (DRM) of which entries are demand to demand elasticities. In the DRM, the phenomena that load reduce at one time and recover at another can be characterized by load redistribution

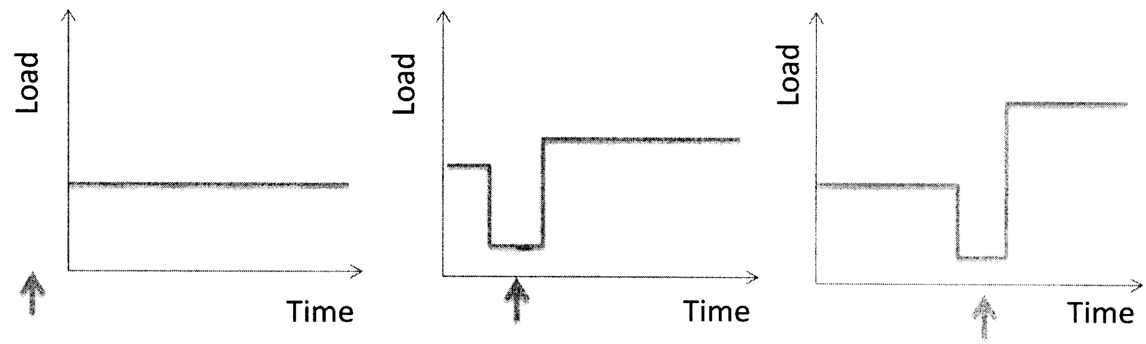


Fig. 3.3.4 Notification timing of incentive-based demand response programs on load profiles.

Transformations of the PEMs

The proposed bidding mechanism can be applied to more occasions with the PEMs' transformations. These occasions include: in RT markets, for industrial end users, under production management and for end user sensitive to other factors.

In RT Markets:

In RT markets, no bids are submitted or accepted for lack of transition time. The generation units and demand directly respond to the RT price released. In traditional bidding mechanism, the ISO calculated the RT prices based on generation bids of DA market, updated demand forecasts and real-times conditions in ancillary markets. In the proposed bidding mechanism, however, since the DA prices are calculated considering demand response and thus at the market equilibrium, there is no need to update demand forecasts in real time. Nevertheless, the demand curve may change due to exogenous factors such as weather. For this reason, fine adjustment of PEMs' entries and of reference points are needed to compute the optimal RT prices.

Moreover, end users cannot reschedule the electric use of the hours before the RT is released. Therefore, assume the RT price is calculated at hour τ, then the ISO should update the PEM by setting its columns before τ to zero. The final payments of the DA and RT markets are settled by either the two-settlement or the post-settlement system.

For industrial end users

A transformation of the PEM is to Demand Redistribution Matrix (DRM) of which entries are demand to demand elasticities. In the DRM, the phenomena that load reduce at one time and recover at another can be characterized by load redistribution coefficient matrix π. The DRM is easier to describe than the PEM for industrial end users and under incentive-based DR programs.

For production management

Sometimes load relocation takes place in a macro perspective such as from one consumer sector to another sector. It is therefore necessary to extend this to the case when some consumers will alter the supply reliability level which they have selected, and when this happens, a portion of load in one consumer reliability sector will transfer to another. This situation can be represented by:

$$E_{Q\times Q} = \begin{bmatrix} E_{11} & \cdots & E_{1Q} \\ \vdots & \ddots & \vdots \\ E_{Q1} & \cdots & E_{QQ} \end{bmatrix}_{(Q\times Q)},$$

where sub-matrix E_{iq}, represents the effect of the load variation of consumer sectors q on the load of consumer sector i. This matrix $E_{Q\times Q}$ describes constraints in production management. These constraints are invisible to end users who are expected profit-maximizing firms, but should be considered in at the macro level. For this reason, if we consider setting the optimal price in DA markets, $E_{T\times T}$ only appears in formulation (31)'s but not formulation (24)'s constraints. Consequently, the optimal price under $E_{T\times T}$ is:

$$p_i^*(t) = \theta_t \left[1 + \frac{\partial l(t)}{\partial D_i(t)}\right] + \sum_k \frac{\partial Z_k(t)}{\partial D_i(t)} \eta_{k,t} + \sum_{q\neq i} E_{i,q}\phi_{q,t} + \left(1 - E_{i,i}\right)\phi_{i,t},$$

which has a structure as:

optimal price to i = [social cost of additional demand at the swing bus]

× [1+ incremental losses caused by i]

+ [transmission constraint terms, summed over lines]

+ [production management constraint terms, summed over related sectors]

The first two terms of the optimal price have the same physical meaning defined in Section 3.3.2. The last term is the marginal demand cost to the system caused by production management constraints.

Other factors:

Inter-temporal constraints on demand and system sides can be described in elasticity matrices. These transformed PEM can be applied in the proposed bidding mechanism with the original PEM. The bidding rules, bidding acceptance rules and price settlement rules are unchanged with application of these transformed PEMs. However, we should notice that if the transformed PEM describes constraint on demand side, then it should appear in both the demand constraints (formulation (31)) and the system constraints (formulation (24)); if the transformed PEM described constraint on system side, then should appear only in the demand constraints (formulation (31)).

3.3.3 Estimation of the PEM

Demand-side factors affect the establishment of demand curve, and thus affect the establishment of PEM entries' values, which are normalized end-user elasticities. Economics defines price elasticity as consumers' sensitivity to price changes. Normalized elasticities are calculated as the ratio of the absolute change in demand to the absolute change in price, $\epsilon = \frac{\partial d}{\partial p}$ (MW/$). In the proposed bidding mechanism, it is the retailers' responsibility to estimate the PEMs and submit them in bids.

PEM entries' values can be estimated with multiple methods. The most common method is end user survey. This method is time-consuming, high-cost and not very reliable. Another method is to regress demand curve with past data, and derive the PEM from the obtained demand curve by doing partial differentiation. The advantage of this method is that it is faster and getting more accurate with more data accumulated in the regression. Learning effect can be applied to the regression if other exogenous disturbances, such as weather, exist. The disadvantages of this method are: it can be very complex to regress a multi-dimensional function (not to mention in the DA markets the dimension can be 12 or 24); and it will be difficult to do the regression at the beginning with few data available. The most feasible method of PEM estimation is to build mathematical models on end-user electric use. This method requires investigations into the loads' physical characteristics and thus can be very expensive. However, it can be done without too much effort to the industrial consumers and for some load types. For example, the PEM of distributed generators can be estimated easily if we know their capacity and cost function, since they are only operated when the market prices are higher than their marginal cost.

A further issue that needs to be considered is the range of price and demand variation under which the PEM entries remain constant. Some studies argue that if the demand at a certain hour varies far from its normal operating range, end users are likely to become less sensitive to price variations at other hours.[13] Other studies state that end users are less sensitive to prices which vary far from their normal rating range. In addition, there exists a level beyond which load reductions become very difficult and the loads can be considered as inelastic. Furthermore, customers are much less likely to increase or reorganize their production to increase their consumption of electricity in the case of a short-term price drop than they are to react to a price increase [29]. In all above cases, the PEM entries are no longer a constant, but a function depending on demand, price, the changes of demand or price, or other factors. An example illustrating such an occasion is depicted in figure 3.3.5.

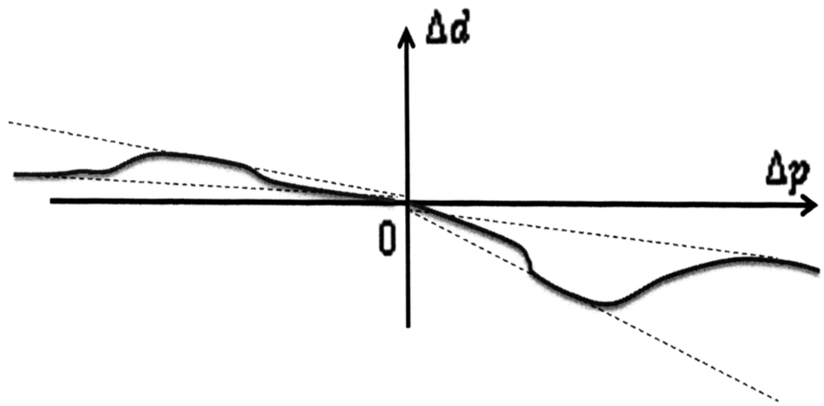
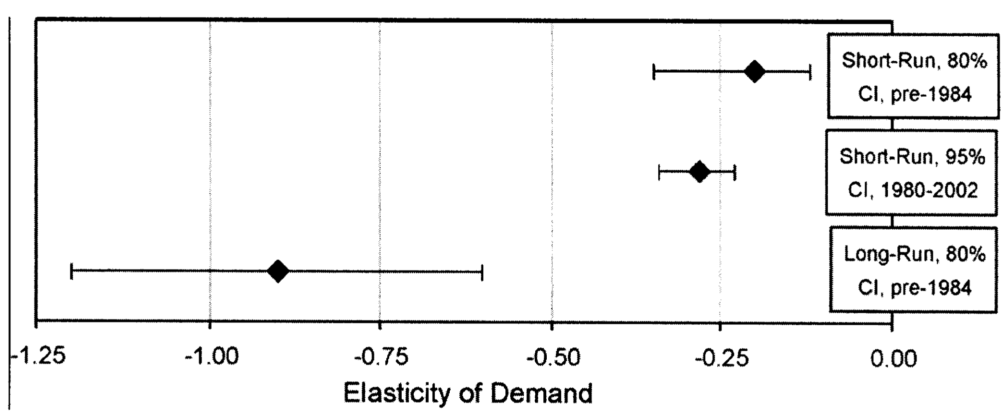
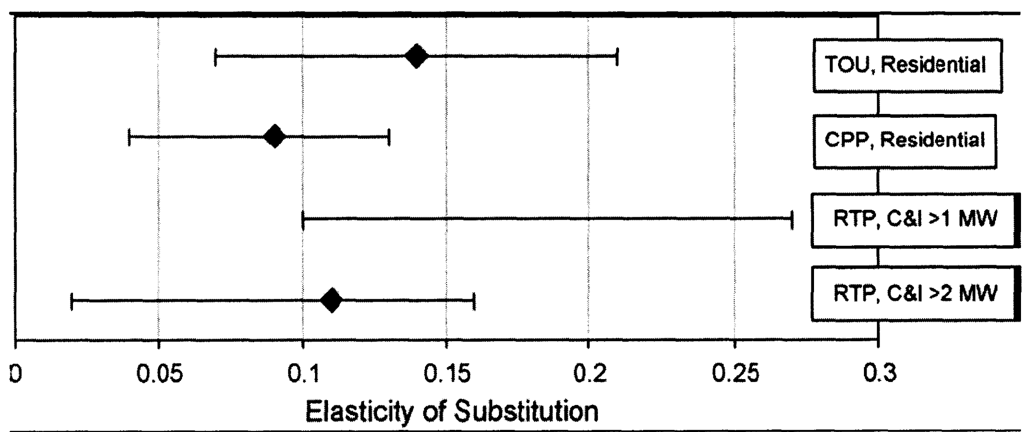


Fig. 3.3.5 the PEM entry varies with demand and price changes. The horizontal axis represents the change of price, and the vertical axis represents the change of demand. The ratios of Δd to Δp of any points on the red segments are the PEM entry's value. Assume the price reference point is \$0, we can observe that the end user is less sensitive to the price change when the price is too low or too high.

Many analyses and experiments have been under taken in order to examine price responsiveness as well as the responsiveness to shifting demand to a lower cost hour. Some experiments are more relevant to demand response because they examine responsiveness to day-ahead hourly prices or with enabling technology. Results are highly variable, partly because responsiveness behavior is complex and highly dependent on the details of the experiment including how prices are communicated. For example, if customers are recruited into a program by being assured that they would not have to pay a higher bill than if they had not participated in the experiment, their incentives are eroded. Similarly, if they know the program will last for only a year or two, they have little incentive to replace appliances or make a capital expenditure that would pay off under a long-term program. Price responsiveness is much greater when customers have an incentive to react by purchasing more efficient appliances and equipment; in the short run, end users can reduce usage only by forgoing or shifting consumption. A 1984 review of 34 short-run and long-run estimates found median elasticities of -0.20 and -0.90, respectively, implying that a 10 percent price increase would reduce consumption by 2 percent in the short run and 9 percent in the long run. Over the long run, these same customers can make additional choices about buying efficient appliances and equipment. Figure 5 shows the difference between short-run and long-run responsiveness [31-33].



A recent Department of Energy review published price elasticities of substitution under TOU, critical peak pricing (CPP), and day-ahead real-time price (RTP) situations. Figure 6 shows averages and ranges reported from four of these studies in residential and commercial and industrial (C&I) sectors. The range of elasticities of substitution was 0.02 to 0.27 [34, 35].



In the future, short-run price elasticity and elasticity of substitution will depend on the sophistication of enabling technology. Modern electronics allow customers to respond to each price change without further thought or effort by having an "energy manager" run electric hot water heaters, dishwashers, pool pumps, and air conditioners during less expensive hours [36].

3.4 Algorithm

This section will briefly discuss the algorithm of the proposed bidding mechanism. One algorithm proposed firstly by David [13] is simulating a market interaction procedure between supply and demand, illustrated in figure 3.4.1.

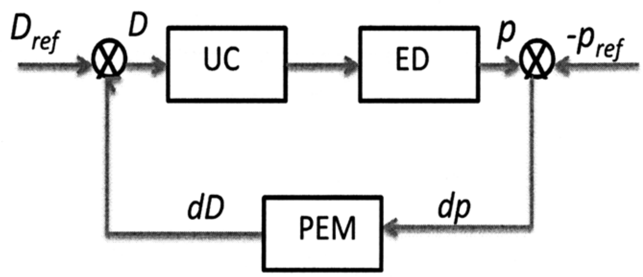


Fig. 3.4.1 the market interaction algorithm. In the figure, the UC block refers to the unit commitment computation, and the ED block refers to the economic dispatch. The PEM block refers to the multiplication operation with the end-user PEM. D and p are the demand and price. D_{ref} and p_{ref} are the reference points of demand and price. dD and dp are the deviation of demand and price from their reference points.

The algorithm can be described by its iterations: in the first iteration, we compute the optimal price and generation schedule with the initial value of consumption $D_0(t)$ in equation (10). Assuming the optimal price obtained is denoted as $p^0(t)$, we compare $p^0(t)$ with the reference price $p^0_{ref}(t)$ and get their difference denoted as $dp^0(t)$. By multiplying the price difference $dp^0(t)$ with the end-user PEM, we get the demand deviation from its reference point $dD^0(t)$. The sum of $D_{ref}(t)$ and $dD^0(t)$ gives us the end-user response to price $p^0(t)$, which is denoted as $D^1(t)$; in the second iteration, we use $D^1(t)$ to compute the optimal price and generation schedule, and repeat the procedure of the first iteration, and start the third iteration... It is predicted that after doing the iterations K times, we can find a market equilibrium, at which $D^K(t)$ equals $D^{K+1}(t)$.

This algorithm can be extended to accommodate multiple retailers participating in the proposed bidding mechanism. More bids from retailers can be included by simply adding feedback branches in parallel, as shown in figure 3.4.2.

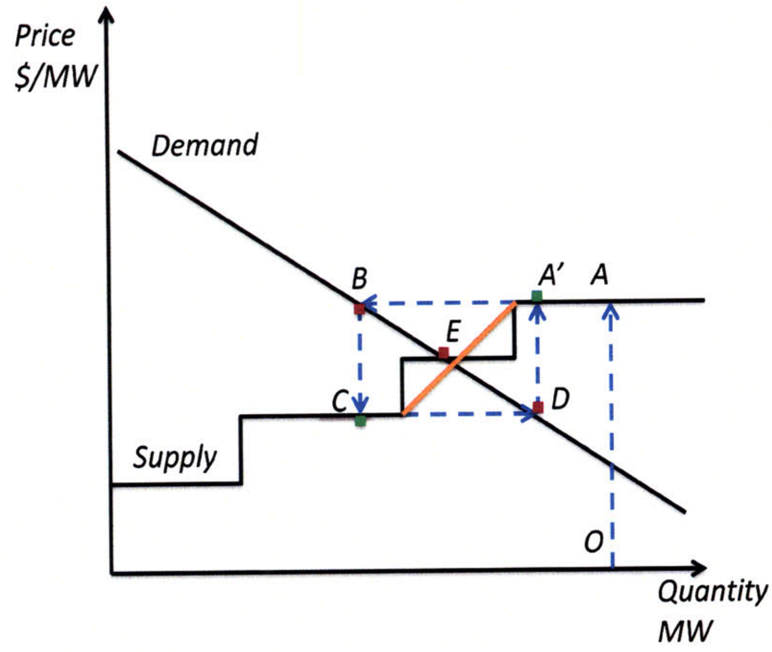


Fig. 3.4.3 Non-convergence caused by steep local relative slope of the supply curve.

This disadvantage is illustrated with a single hour case in figure 3.4.3: If we start from point O, D_O will generate an optimal price p_A in the traditional bidding mechanism; Price p_A triggers the end-user response as D_B; Demand D_B generates an optimal price p_C which in turn triggers the end-user response as D_D. However, since demand D_D gives the original price p_A, the following iterations will be stuck in the four points A', B, C and D but will be never able to reach the market equilibrium E. The cause of this problem is that the demand response from B to D is larger than the total capacity of the generation units of which the marginal costs are between p_B and p_D. Therefore, we can conclude that *the algorithm stops evolve forward to the market equilibrium when the demand curve is not locally sufficiently "steep"*.

This non-convergence condition can be described by the "relative slope" concept that is developed in our research. Using the single-hour example of figure 3.4.3, we define the two generation units between which the non-convergence happens as G_A and G_C. The non-convergence condition is,

$$\underline{cap}(G_A) - \overline{cap}(G_C) \geq \underline{cap}(G_A) - D_k(\lambda(G_A)), \quad (50)$$

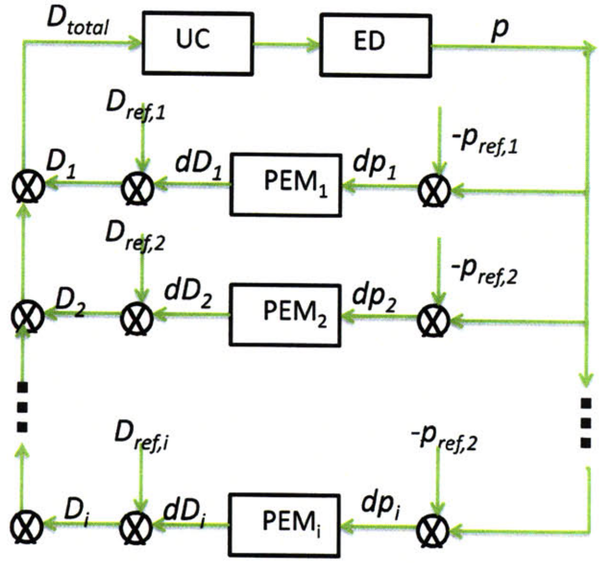


Fig. 3.4.2 The market interaction algorithm with multiple end-user types.

As defined in Section 2.3, the bidding acceptance and price settlement are implemented through running unit commitment and economic dispatch in electrcity market. This process is defined as market clearing process. The key idea behind this algorithm is to first clear market under inelastic demand, and then compute the demand response to the market clearing price. In other words, the algorithm consists of two parts: the market clearing process under traditional bidding mechanism in which the demand is regarded as inelastic, and demand response quantified by the PEM. Repeating the two-part market interaction in every iteration, the price and generation schedule will evolve to the market equilibrium.

3.4.1 The Disadvantages of the Market Interaction Algorithm

The algorithm evolution failure and relative slope

The advantage of this algorithm is simple and easy to understand. However, this algorithm has several disadvantages due to the "interaction" approach. One disadvantage of this algorithm is that it can not guarantee to find a convergent solution even if the market equilibrium exists.

where	$\underline{cap}(G_A)$	the generation capacity summation from the least expensive generation unit to the generation unit that has a marginal cost only less than G_A;
	$\overline{cap}(G_C)$	the generation capacity summation from the least expensive generation unit to G_C;
	$\lambda(G_A)$	the marginal cost of G_A;
	$D_k(p)$	the demand curve function with slope k and independent variable as price p.

Figure 3.3.8 shows that

$$\underline{cap}(G_A) - \overline{cap}(G_C) = \text{ctan}\,\angle DBA \cdot \left(\lambda(G_A) - D_k^{-1}\left(\underline{cap}(G_A)\right)\right), \quad (51)$$

where	$\lambda(G_A)$	the marginal cost of G_A;
	$D_k^{-1}(d)$	the inversed function of demand curve D_k with independent variable as quantity d.

From equation (50) and (51) we have the non-convergence condition as,

$$\text{ctan}\,\angle DBA \cdot \left(\lambda(G_A) - D_k^{-1}\left(\underline{cap}(G_A)\right)\right) \geq \underline{cap}(G_A) - D_k(\lambda(G_A))$$

$$\tan\angle DBA \leq \frac{\lambda(G_A) - D_k^{-1}\left(\underline{cap}(G_A)\right)}{\underline{cap}(G_A) - D_k(\lambda(G_A))}, \qquad (52\ a)$$

where $\tan\angle DBA = \tan\angle BDC$, and $\tan\angle BDC$ is defined as the slope k of the demand curve D_k. Therefore, we rewrite equation (52 a) as

$$k \le k_{A,C} \qquad (52\ b)$$

and

$$k_{A,C} = \frac{\lambda(G_A) - D_k^{-1}\left(\underline{cap}(G_A)\right)}{\underline{cap}(G_A) - D_k(\lambda(G_A))}, \quad (52\ c)$$

where $k_{A,C}$ is the *relative slope* of the step-sized supply curve segment defined by G_A and G_C. Given this relative slope $k_{A,C}$, we can linearize the step-sized supply curve segment from G_A to G_C as the orange line segment in figure 3.4.3. For multi-period cases, load shifting effects caused by other hours may induce unconvergent behavior as well. Therefore, equation (52 b) and (52 c) are the sufficient but not necessary condition. To apply this condition in multi-period cases, the demand curves' slopes are set as the self-elasticities of the PEMs. The relative slopes of the supply curves can be found between any two generation units and using equation (52 c), and thus determines the convergence under the market interaction algorithm.

The algorithm evolution failure and demand clears the market

Another disadvantage is observed when the demand clears the market, shown in figure 3.4.4. In the proposed bidding mechanism, when demand is on the brder of the marginal unit or exceeds the total generation, we allow demand to clear market. In fact, demand-clearing market generates the curtailment premium μ, defined in equation (28), which indicated the system marginal cost of starting another generation unit. Demand, since considered as inelastic, is not allowed to clear market in traditional bidding mechanisms. In traditional bidding mechanisms [10], when the market is required to be cleared by demand is defined as market equilibrium not existing. In the situation illustrated in figure 3.4.4., the mentioned algorithm can never reach the market equilibrium, since clears market under traditional bidding mechanism E. The results of iterations will cycle among the market equilibrium's neighborhood, A, B, C and D. Therefore, with the mentioned algorithm undermines the benefit of the proposed algorithm, since we cannot find the market equilibrium nor calculate the curtailment premium when demand is needed to clear the market.

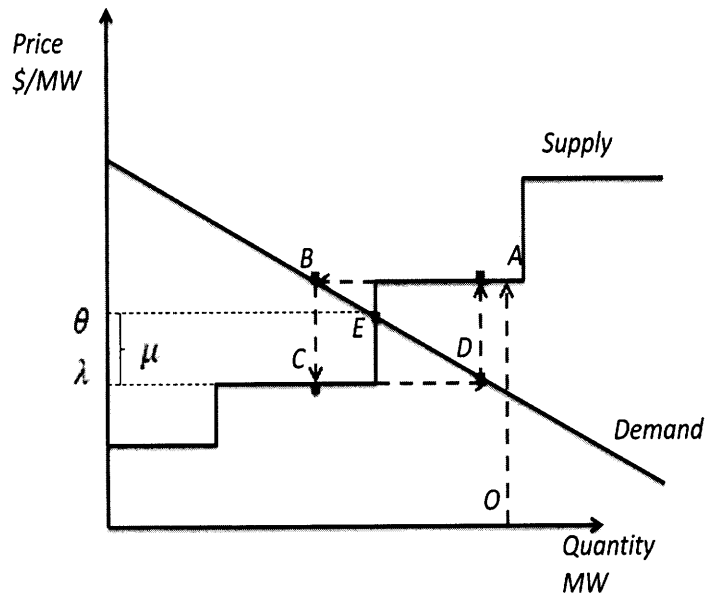


Fig. 3.4.4 Non-convergence caused by demand clears the market.

3.4.2 The Improved Market Interaction Algorithm

I improve the mentioned algorithm by considering the two mentioned situations in figure 3.3.9 and 3.3.10 whenever results begin fluctuating between several values. More delicate algorithm can be developed based on formulation (40), but it is out the scope of this thesis. The diagram of the improved algorithm is shown in figure 3.4.5.

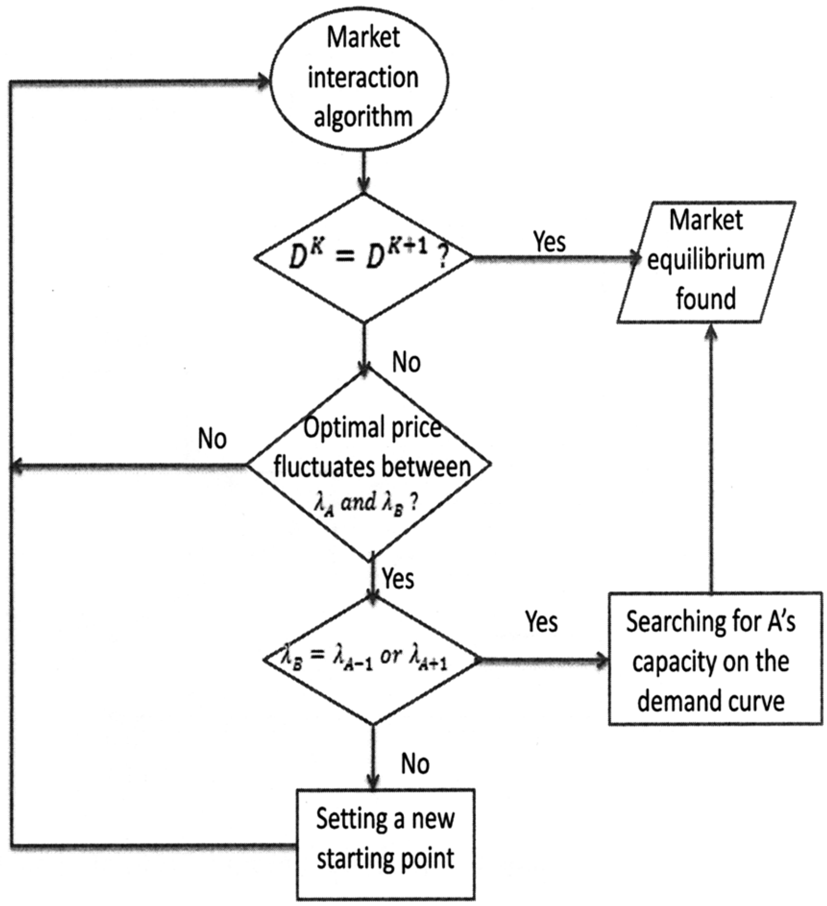


Fig. 3.4.5: The improved market interaction algorithm.

To find the market equilibrium, we first run the market interaction algorithm. For every iteration K, we check if the market equilibrium is found by checking if $D^K = D^{K+1}$ holds. If the optimal prices of the recent iterations oscillate among several values, for simplicity, say between $\lambda(G_\alpha)$ and $\lambda(G_\beta)$, we further examine the reason for the oscillation. The oscillation is caused by demand clears the market if the generation unit G_β has the marginal cost ordered next to G_α. An alternative is to compare the demand curve slope with the local relative slope of the supply curve by equation (52 b) and (52 c). The oscillation is caused by the relative slope of the supply curve is locally greater than the slope of the demand curve.

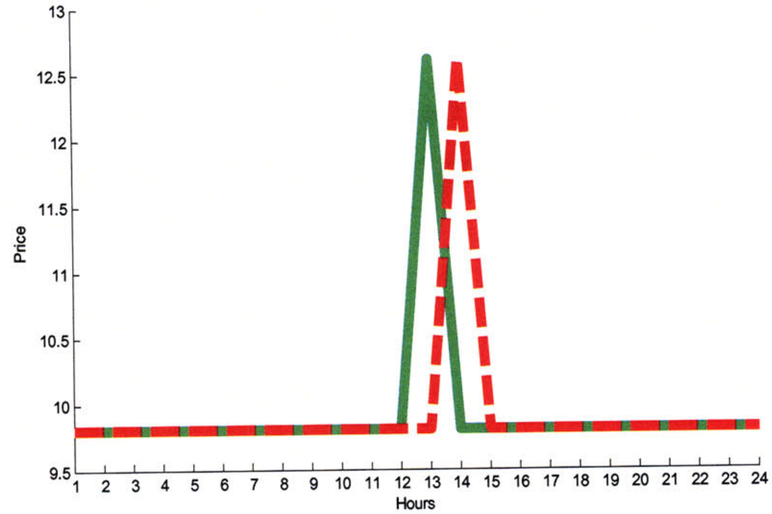


Fig. 3.4.7 Price ocsillation happens at multiple hours. The two curves are the market clearing prices obtained from two sequential iterations of the market interaction algorithm. In this figure, prices oscillate between 9.8 and 12.6 at Hour 13 and Hour 14. Assuming 9.8 and 12.6 are marginal costs of generation units G_α and $G_{\alpha+1}$, we know that price may clear the market at both Hour 13 and Hour 14. However, it is possible that only the price at Hour 13 clears the market and the oscillation at Hour 14 is due to the load shifting from Hour 13, or the other way around. Except for Hour 13 and Hour 14, the prices converge to 9.8 at all the other hours.

It is because the price oscillation between $\lambda(G_\alpha)$ and $\lambda(G_{\alpha+1})$ or $\lambda(G_{\alpha-1})$ at hour H_n may be due to the demand clears the market at H_n, or it can be due to the load shifting from other hours $H_{\tilde{n}}$ at which demand clears the market. For this reason, when prices oscillate between $\lambda(G_\alpha)$ and $\lambda(G_\beta)$, where $\lambda(G_\beta)$ equals to $\lambda(G_{\alpha+1})$ or $\lambda(G_{\alpha-1})$, at multiple hours, we search on the demand curve for the market equilibrium according to the following steps:

1. Among all possible mutations, denote all the hours at which prices oscillate between $\lambda(G_\alpha)$ and $\lambda(G_\beta)$ as H_1. Among H_1, denote the hours at which demand clears the market as H_{DCM} and other hours as $H_{\widetilde{DCM}}$; Denote all other hours at which prices are convergent as H_0;
2. At H_{DCM}, set prices as variables p_{DCM}, and set demand quantity d_{DCM} as the lower quantity of $\overline{cap}(G_\alpha)$ and $\overline{cap}(G_\beta)$; At $H_{\widetilde{DCM}}$, set prices $p_{\widetilde{DCM}}$ as

The point setting method

When oscillations are resulted from the supply surface is locally too steep, we reverse the evolving direction of the algorithm by setting another demand starting point. Figure 3.4.6 illustrates this idea.

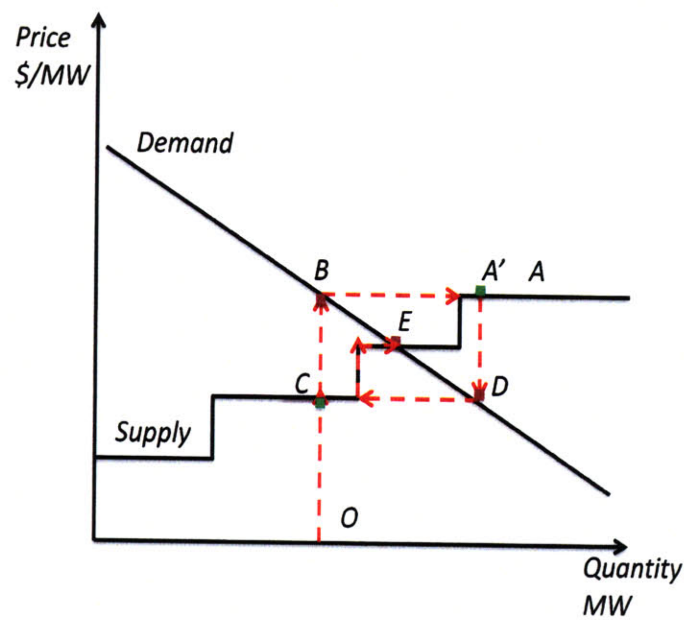


Fig. 3.4.6: Searching for the market equilibrium under non-convergence. Reverse the searching direction when the local relative slope of the supply curve is steeper than the slope of the demand curve.

The demand curve searching algorithm

When demand clears the market, we search on the demand curve for the market equilibrium. Consider the single hour market interaction in figure 3.3.9. The market should be cleared at the demand that equals to the lower capacity of unit G_α; or G_β, $\overline{cap}(G_C)$ in figure 3.3.9. Therefore, we can find on the demand curve for the market clearing price at the point of which the demand is $\overline{cap}(G_C)$. In multi-period market interaction, the searching can become much more complex. Figure 3.4.7 gives an example.

$\lambda(G_\alpha)$ or $\lambda(G_\beta)$, and set demand quantity as variable $d_{\overline{DCM}}$; At H_0, set prices as their convergent prices p_0, and set demand quantity as variable d_0;

3. Substitute all parameters and variables into equation (15), which is

$$\begin{bmatrix} \Delta d_1 \\ \Delta d_2 \\ \vdots \\ \Delta d_T \end{bmatrix} = \begin{bmatrix} \epsilon_{11} & \epsilon_{11} & \cdots & \epsilon_{1T} \\ \epsilon_{21} & \epsilon_{22} & \cdots & \epsilon_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon_{T1} & \epsilon_{T2} & \cdots & \epsilon_{TT} \end{bmatrix} \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \vdots \\ \Delta p_T \end{bmatrix}, \quad (15)$$

where $\Delta d_t = d_t - d_{ref,t}$, $\Delta p_t = p_t - p_{ref,t}$ and $H_1 \cap H_0 = \mathcal{T}$. We obtain $|\mathcal{T}|$ linear equations to solve $|\mathcal{T}|$ variables p_{DCM}, $d_{\overline{DCM}}$ and d_0;

4. Check the solution of equation (15). The solution is feasible if:
   a. $p_{DCM} \in (\lambda(G_\alpha), \lambda(G_\beta))$ and
   b. $d_{\overline{DCM}} \leq \overline{cap}(G_\alpha)$ if $p_{\overline{DCM}}$ is set as $\lambda(G_\alpha)$; $d_{\overline{DCM}} \leq \overline{cap}(G_\beta)$ if $p_{\overline{DCM}}$ is set as $\lambda(G_\beta)$.
5. The searching algorithm terminates when the feasible solution is obtained. Otherwise, go back to step 1 and try another mutation of H_{DCM} and $H_{\overline{DCM}}$ in H_1.

For H_0, the variables d_0 are predicted to always fall in the range that induces the convergent price p_0. Because if the shifting effects from prices $\lambda(G_\alpha)$ and $\lambda(G_\beta)$ both result in the demands, $d_{0,\alpha}$ and $d_{0,\beta}$, that induce p_0, then any price $p_{DCM} \in (\lambda(G_\alpha), \lambda(G_\beta))$ will result in a demand, $d_{0,DCM} \in (d_{0,\alpha}, d_{0,\beta})$, that induces p_0 as well.

This searching algorithm terminates once a feasible solution is found. In other words, a unique solution is expected among all candidate solutions, which are generated by mutating H_{DCM} and $H_{\overline{DCM}}$ in H_1. This is based on the fact that the market has a unique equilibrium if the supply curve and the demand curve are both linear.

Chapter 4

Numerical Examples

An essential characteristic of the proposed bidding mechanism is demand response to RT price and inter-temporal load shifting effects. This characteristic results in multiple benefits such as reducing RT load uncertainties, increasing power systems' reliability and reducing the cost of balancing spot electricity market.

In order to illustrate this characteristic and the working process of the proposed bidding mechanism, this chapter gives sets of numerical examples under various systems and loads' conditions. Section 4.1 describes the simulation environment and data background. Section 4.2 presents and analyzes the numerical examples. The full case description and raw data of the numerical examples are presented in Appendix A.

4.1 Simulation Environment and Data Background

The numerical examples in this chapter are classified into five sets according to their sub-market environment types, systems' status and end-user types.

The entire market of this thesis is wholesale DA and RT energy-trading electricity pools. For the convenience of data presentation, the sub-market types under which the numerical examples are simulated are set as the DA and Hourly Ahead (HA) markets. The HA market is the same as RT market in the perspectives of the bidding acceptance rules and the price settlement of the proposed bidding mechanism. The only difference of the HA and RT markets is that the RT market has a much short transaction period, usually as five minutes, and thus requires much larger sized PEMs and more computation repetitions to settle down the final payment. This statement is illustrated in the numerical examples set V.

In the numerical examples' simulation, we consider three major system states: the ordinary state, under contingencies and with renewable energy generation. The contingent state covers two major type contingencies in power systems: sudden loss of transmission and sudden change of generation units' marginal cost. The involvement of renewable energy brings a stochastic factor from the generation side. The numerical example shows that the proposed bidding mechanism has more benefits under these two unordinary system states.

Nine end-user types are examined in the numerical examples. These end-user types are modeled by PEMs and load operation constraints. Shifting effects are observed and analyzed in perspectives of generation dispatch schedules, prices and final load profiles.

The system status, end-user types and sub-market types together define the settings of a simulation case. Every simulation case examines two sets of bidding data. The first data set covers bids of five hourly periods. Demand bids of this data set have flat reference values in equation (15). The numerical examples obtained from this data set are more observable in end-user shifting patterns. The second data set contains bids of 24 hourly periods. The demand bids are deduced from the load data of New York Independent System (NYISO). The load data are collected from power transactions of New York City, Dunwod and Long Island in August 09, 2009. Figure 4.1.1 shows the control region map of NYISO. The numerical examples obtained from this data gives an intuition of the practical performance of the proposed bidding mechanism.

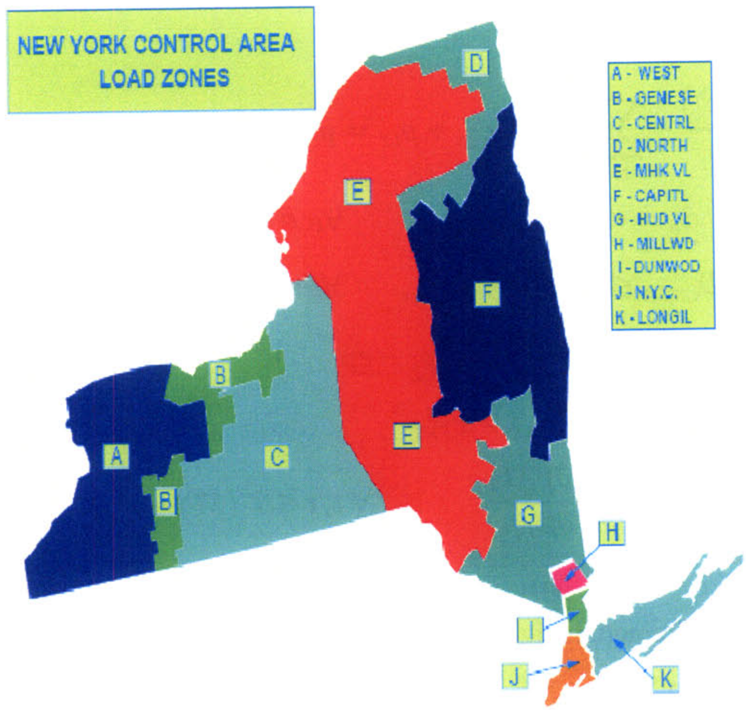


Fig. 4.1.1 Control region map of the NYISO.

The numerical examples are examined in a system with three generation units and three retailers. For data analysis convenience, no network constraints are considered, and only the capacity limits are considered among all generation constraints. Therefore, the formulation of the numerical example in this chapter is,

$$\text{max: } \sum_t \sum_i F_i(D_i(t)) - \sum_t \sum_j \lambda_j Y_j(t), \quad (40)$$

s.t.

Generation constraints:

$$0 \leq Y_j(t) \leq K_j, \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (16)$$

Demand constraints:

$$\frac{\partial F_i(D_i(t))}{\partial D_i(t)} + \sum_n \frac{\partial d_{e,n}}{\partial D_i(t)} v_{e,n} + \rho_{i,t} = p_i(t), \;\; \forall i \in \mathcal{I}, t \in \mathcal{T} \quad (34)$$

$$s.t. \; \underline{d_e} = < d_{e,1}, \ldots, d_{e,n} >, \qquad \forall e \in \mathcal{E}, n \in \mathcal{N}_e \quad (32)$$

$$D_{i,min} \leq D_i(t) \leq D_{i,max}, \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \; (33)$$

$$\underline{D_i} - \underline{e}D_{i,ref} = \varepsilon_{T\times T}\left(\underline{p_i} - \underline{e}p_{i,ref}\right), \qquad (15b)$$

$$\xi_{T\times T} = \varepsilon_{T\times T}^{-1}, \qquad (36)$$

$$\xi_{t\tau} = \frac{\partial p_i(t)}{\partial D_i(\tau)}, \qquad \forall \, t, \tau \in \mathcal{T} \; (37)$$

The improved market interaction algorithm is used to solve the above formulation. This algorithm is presented in Section 3.4. The optimization software is the General Algebraic Modeling System (GAMS), module rev 149. The interfaces of data transfer and process are written in MATLAB, version 7.4.0.287 (R2007a).

4.2 Examples under Various End-User Types

This section presents numerical examples of the proposed bidding mechanism's performance under various end-user types. Nine end-user types are examined individually. The nine end-user types are: curtailable load, early end users, late end users, forward shifting end users, backward shifting end users, flexible end users, real world end users, distributed generation and on-site storage.

The numerical examples are examined in a system with three generation units and three retailers. Table 4.2.1 presents bids of the three generation units. For data presentation's convenience, all quantities in this section are normalized into unit numbers. In addition, capacities and marginal costs of generation units are assumed time-invariant in this section.

TABLE 4.2.1 Generation Parameters

Generation No.	Capacity (unit)	Marginal Cost (unit)
G1	1.0	9.8
G2	0.7	10.7
G3	0.5	12.6

This section examines the numerical examples in the DA market. The transaction period of the DA market is set as one hour. The three retailers are considered identical. In other words, they submit the same reference points and load shifting pattern. Two data sets are used as the retailers' bids. The first data set is of five-period timeframe. The reference prices and loads are flat in time, shown in figure 4.2.1.

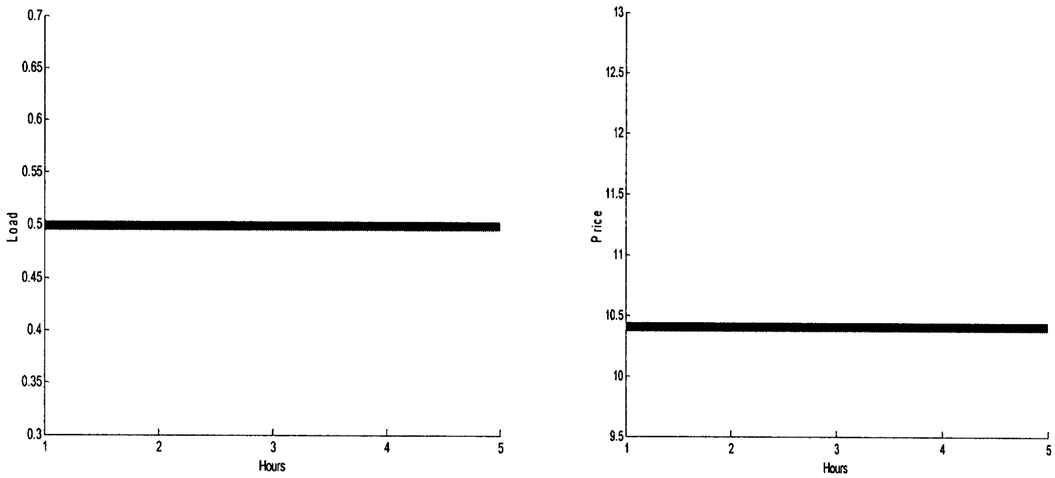


Fig. 4.2.1 Reference load and price of five-period retailers' bids

The second data set is of 24-period timeframe. The reference prices and loads are derived from the electric use of Long Island, New York, in August 9, 2008. The data source is introduced in Section 4.1. Figure 4.2.2 depicts the reference points.

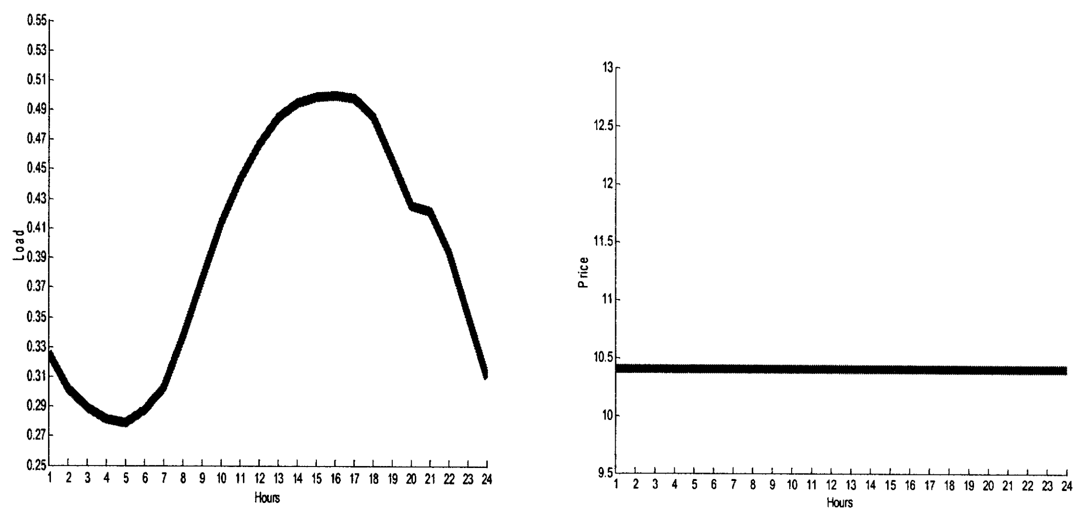
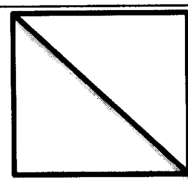
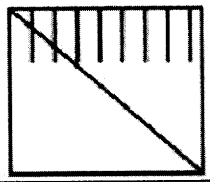
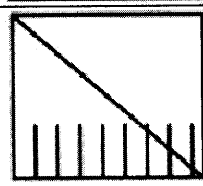
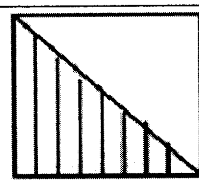
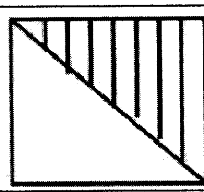
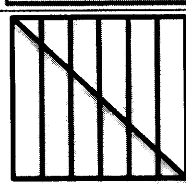
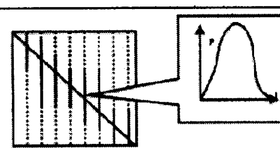
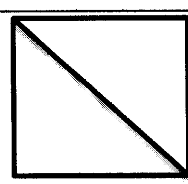
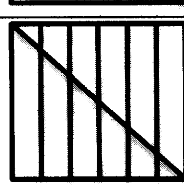


Fig. 4.2.2 Reference load and price of 24-period retailers' bids

The transaction timeframe of the DA market equals to the timeframe of the data set applied. The initial load values are set the same as the reference loads. In addition, we set maximum loads to 1.0 and minimum loads to 0.0.

The nine end-user types are modeled by nine PEMs. Table 4.2.2 row 2 to 10 describes the end-user types and their PEM models for all the numerical examples. The "PEM" column of Table 4.2.2 gives illustrations of the PEMs' topologies of all the end-user types. The last column of Table 4.2.2 gives the PEM examples of the 5-period data

TABLE 4.2.2

No.	End-User Type	PEM	5-Period Example
1	Inelastic	N/A	N/A
2	Curtailable Load		$\begin{bmatrix} -0.2 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.2 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -0.2 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -0.2 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -0.2 \end{bmatrix}$
3	Early End User		$\begin{bmatrix} -0.2 & 0.20 & 0.10 & 0.10 & 0.10 \\ 0.20 & -0.2 & 0.10 & 0.10 & 0.10 \\ 0.00 & 0.00 & -0.2 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -0.2 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -0.2 \end{bmatrix}$
4	Late End User		$\begin{bmatrix} -0.2 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.2 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -0.2 & 0.00 & 0.00 \\ 0.10 & 0.10 & 0.10 & -0.2 & 0.20 \\ 0.10 & 0.10 & 0.10 & 0.20 & -0.2 \end{bmatrix}$
5	Forward Shifting End User		$\begin{bmatrix} -0.2 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.05 & -0.2 & 0.00 & 0.00 & 0.00 \\ 0.05 & 0.067 & -0.2 & 0.00 & 0.00 \\ 0.05 & 0.067 & 0.10 & -0.2 & 0.00 \\ 0.05 & 0.067 & 0.10 & 0.20 & -0.2 \end{bmatrix}$
6	Backward Shifting End User		$\begin{bmatrix} -0.2 & 0.20 & 0.10 & 0.067 & 0.05 \\ 0.00 & -0.2 & 0.10 & 0.067 & 0.05 \\ 0.00 & 0.00 & -0.2 & 0.067 & 0.05 \\ 0.00 & 0.00 & 0.00 & -0.2 & 0.05 \\ 0.00 & 0.00 & 0.00 & 0.00 & -0.2 \end{bmatrix}$
7	Flexible End User		$\begin{bmatrix} -0.2 & 0.05 & 0.05 & 0.05 & 0.05 \\ 0.05 & -0.2 & 0.05 & 0.05 & 0.05 \\ 0.05 & 0.05 & -0.2 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & -0.2 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & -0.2 \end{bmatrix}$
8	Real-World End User		$\begin{bmatrix} -0.2 & 0.06 & 0.03 & 0.01 & 0.01 \\ 0.06 & -0.2 & 0.06 & 0.03 & 0.02 \\ 0.03 & 0.06 & -0.2 & 0.06 & 0.03 \\ 0.02 & 0.03 & 0.06 & -0.2 & 0.06 \\ 0.01 & 0.01 & 0.03 & 0.06 & -0.2 \end{bmatrix}$
9	Distributed Generation		$\begin{bmatrix} -0.2 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.2 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -0.2 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -0.2 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -0.2 \end{bmatrix}$
10	On-Site Storage		$\begin{bmatrix} -0.2 & 0.05 & 0.05 & 0.05 & 0.05 \\ 0.05 & -0.2 & 0.05 & 0.05 & 0.05 \\ 0.05 & 0.05 & -0.2 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & -0.2 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.05 & -0.2 \end{bmatrix}$

set. The end-user types of the 24-period data set are modeled in an identical way. Appendix B presents the code generating the ten 24×24 PEMs.

Two other major traditional bidding mechanisms are compared with the proposed bidding mechanism. The first bidding mechanism considers demand as inelastic, and the second bidding mechanism is the Single Hourly Bidding (SHB) which considers single hourly demand elasticity but ignores the inter-temporal load shifting effects. These two bidding mechanisms are introduced in Section 2.3. Bidding results of the numerical examples are presented in Table 4.2.3 and Table 4.2.4. The case numbers of these two table are consistent with the end-user type numbers in Table 4.2.1. The case number of the inelastic demand bidding mechanism is No. 1, and the case number of SHB is No. 2.

In Table 4.2.3, the reference prices and reference loads are plotted as the red lines in the figures of final load profile and in market clear price.

Case 1 describes the bidding results under the inelastic demand bidding mechanism. The bidding results also appear under the proposed bidding mechanism if the end users are not affected by the prices, i.e. inflexible end users.

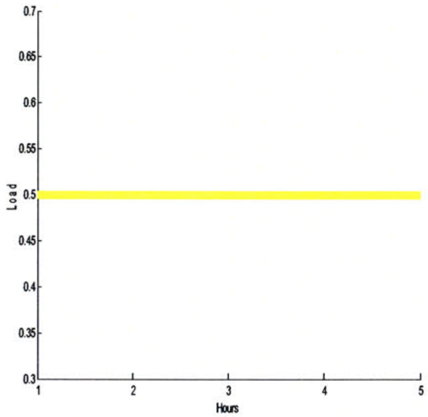
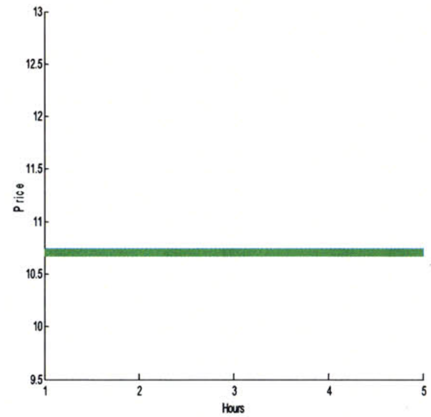
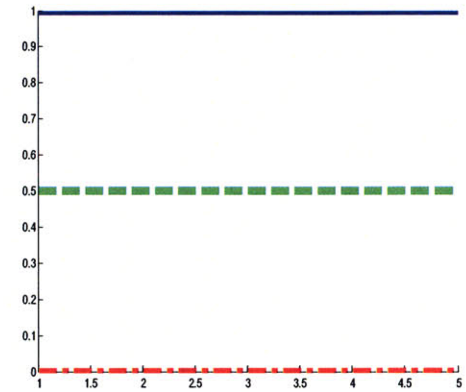
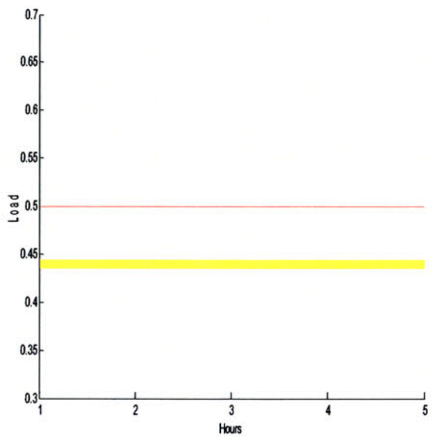
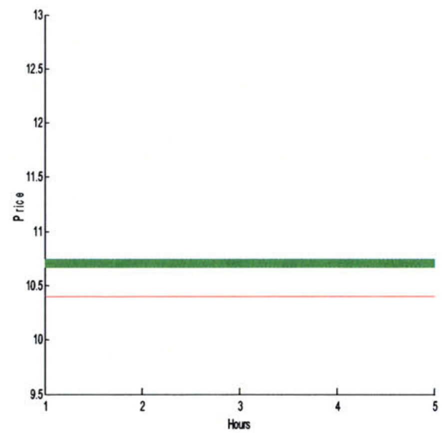
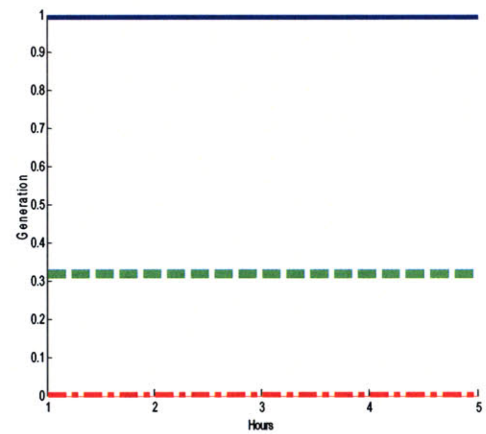
Case 2 describes the bidding results under the SHB. Its results show that the final hourly loads are higher than the reference loads since the hourly prices are higher than the reference prices. The bidding results also appear under the proposed bidding mechanism, if the end-user type is curtailable load. No shifting behaviors is observed in this case.

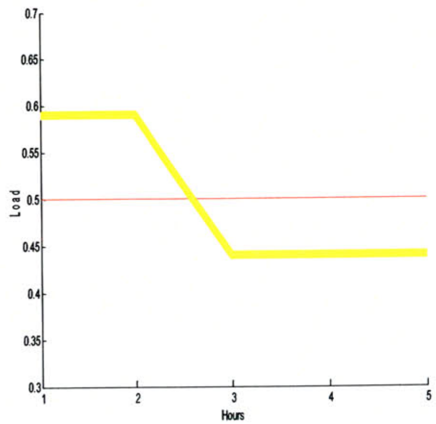
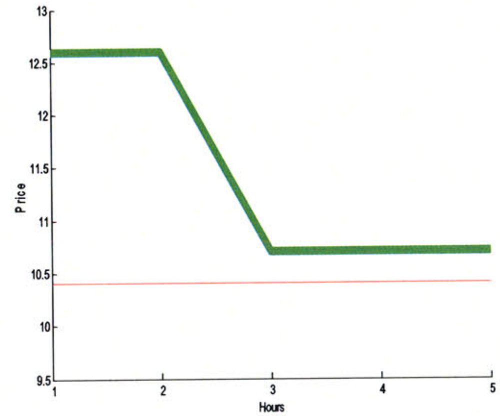
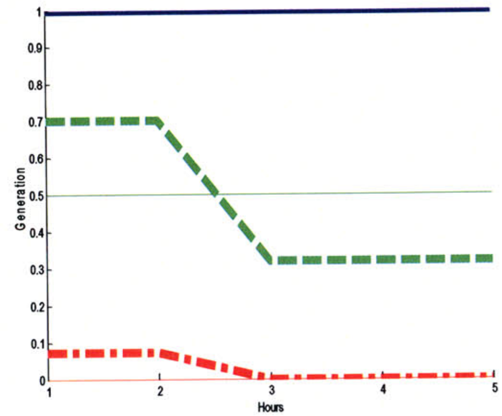
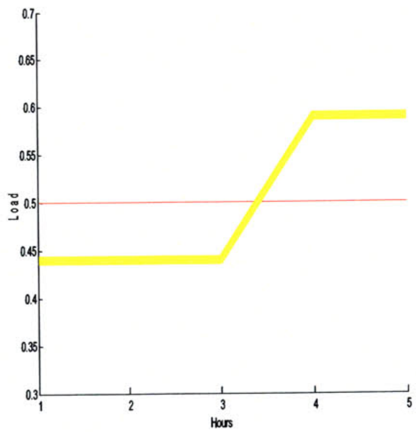
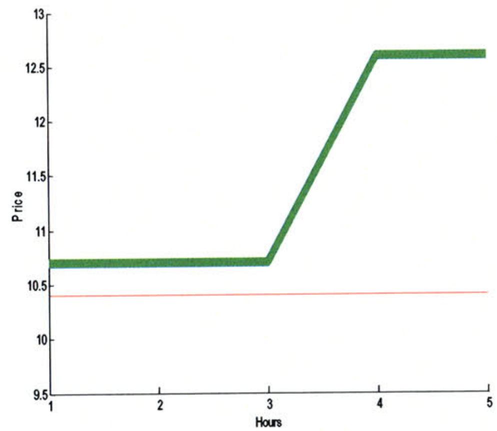
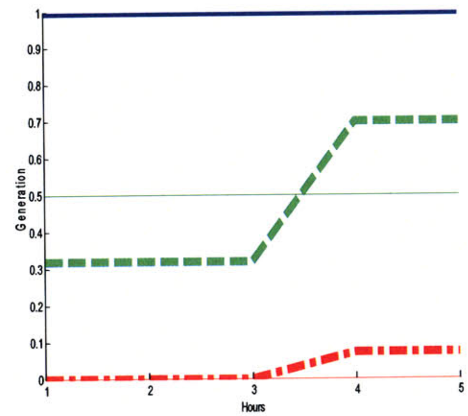
Case 3 and case 4 are early end users and late end users. The results show that the market clearing prices are higher than the reference prices. Therefore, end users shift loads to the early or late hours according to their end-user types. In addition, since their PEMs satisfy the lossless condition in equation (48), the total reduced electric use equals to the increased electric use in the timeframe.

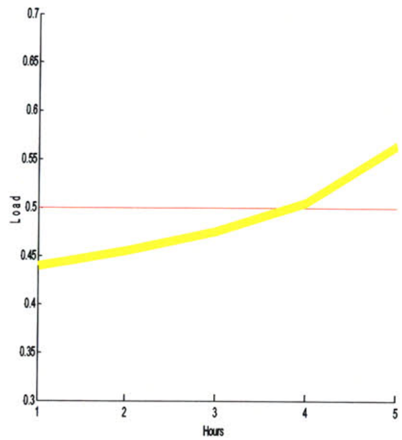
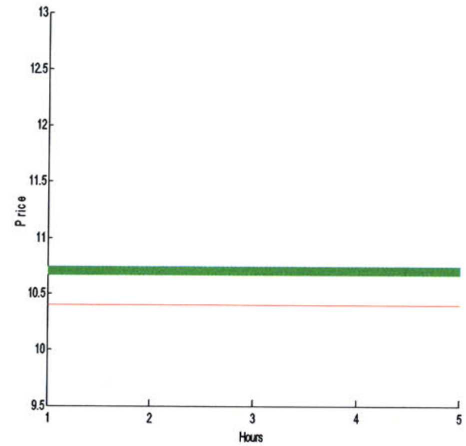
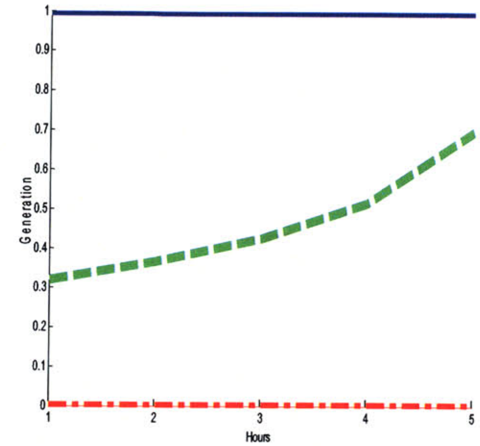
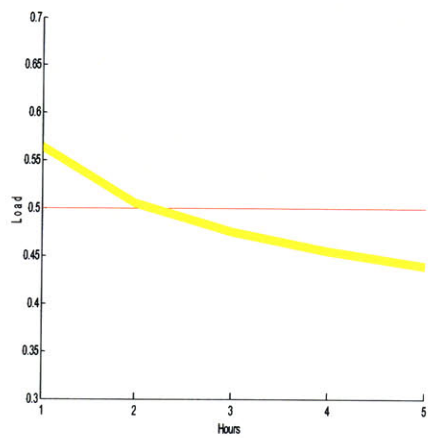
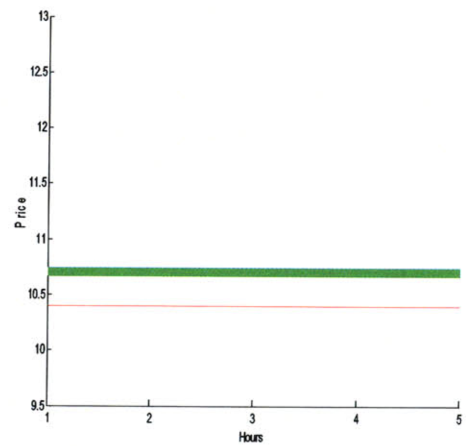
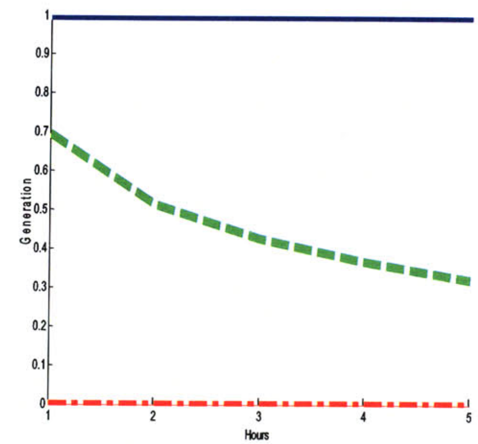
Case 5 and case 6 are forward shifting users and backward shifting users. The results show that the market clearing prices are higher than the reference prices. End users shift forward or backward according to their end-user types.

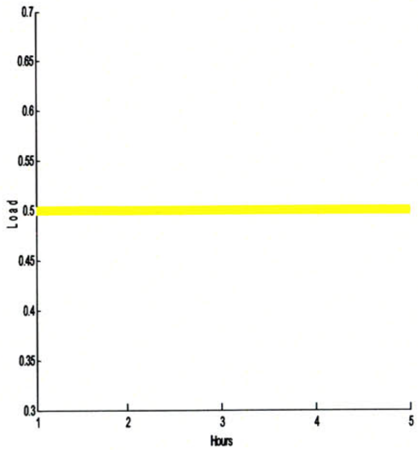
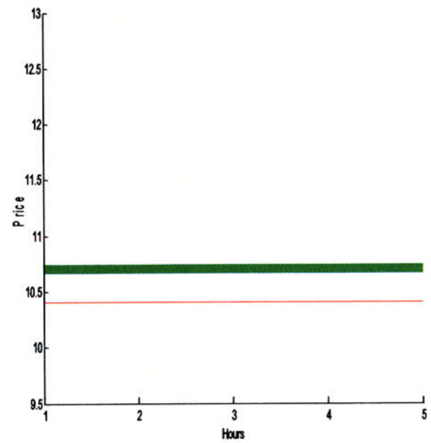
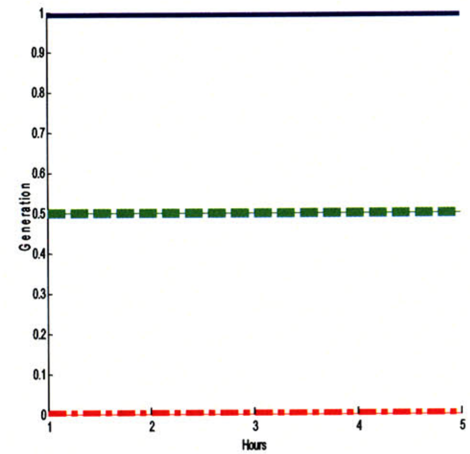
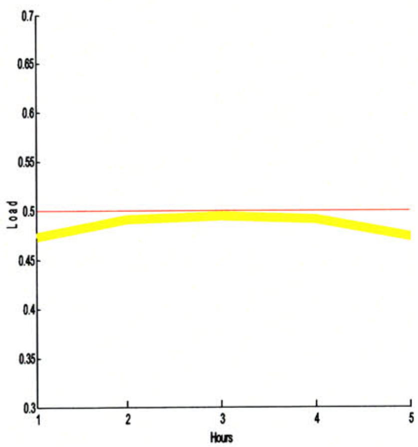
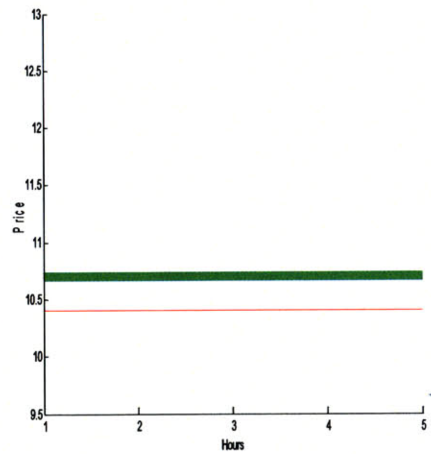
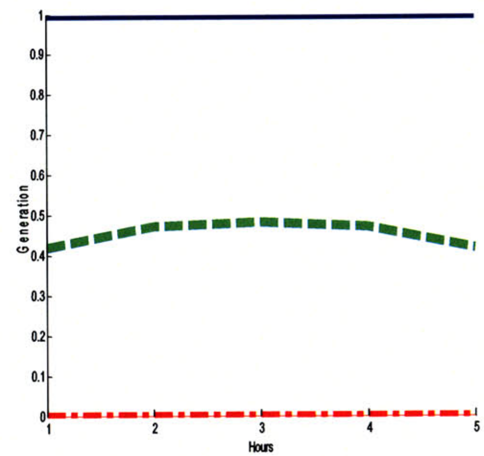
Case 7 is flexible end users. The results are identical to those of case 1. It is because the PEM of case 7 satisfies the lossless condition in equation (48). The hourly shifting effects cancel off each other and result in a flat load profile.

TABLE 4.2.3 Bidding results of the proposed bidding mechanism under nine end-user types (5 periods)

No.	Load Profile	Market Clearing Price	Generation Dispatch Schedule
1			
2			

<table>
<tr><th>No.</th><th>Load Profile</th><th>Market Clearing Price</th><th>Generation Dispatch Schedule</th></tr>
<tr><td>3</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>4</td><td>
</td><td>
</td><td>
</td></tr>
</table>

<table>
<tr><th>No.</th><th>Load Profile</th><th>Market Clearing Price</th><th>Generation Dispatch Schedule</th></tr>
<tr><td>5</td><td></td><td></td><td></td></tr>
<tr><td>6</td><td></td><td></td><td></td></tr>
</table>

No.	Load Profile	Market Clearing Price	Generation Dispatch Schedule
7			
8			

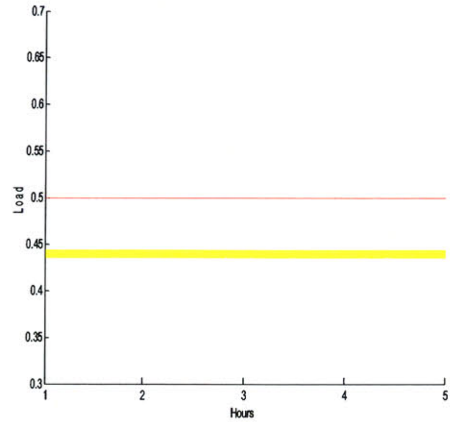
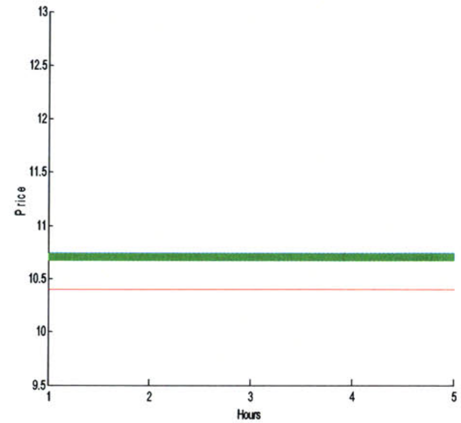
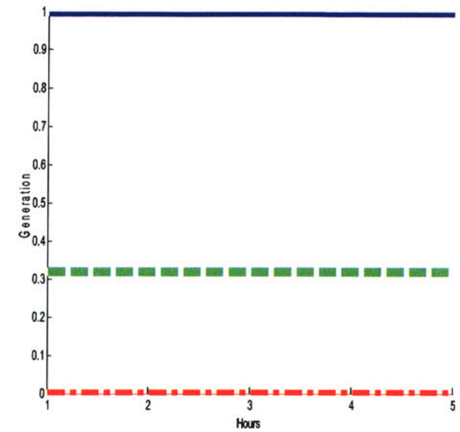
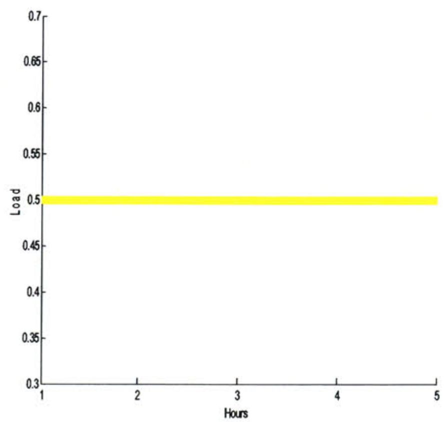
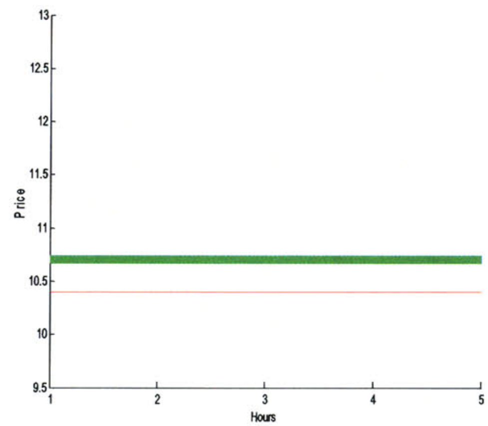
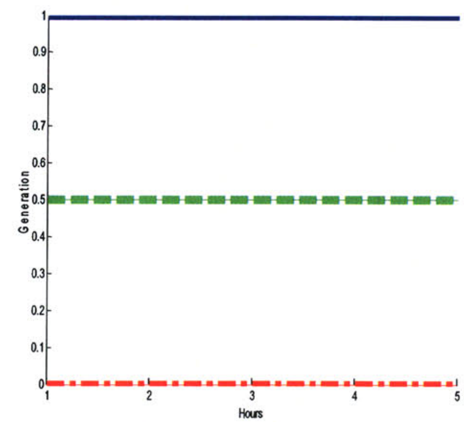
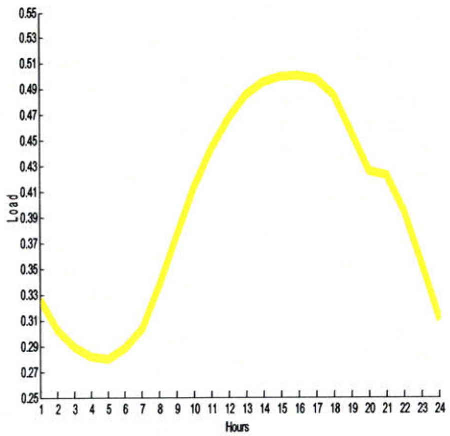
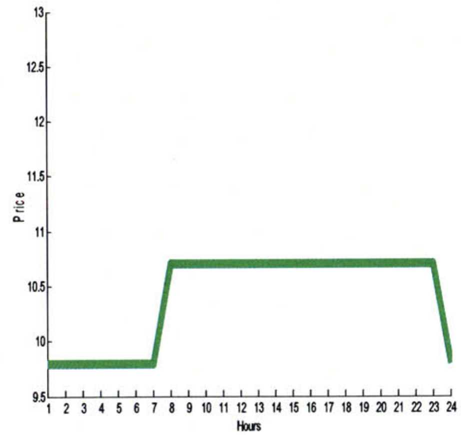
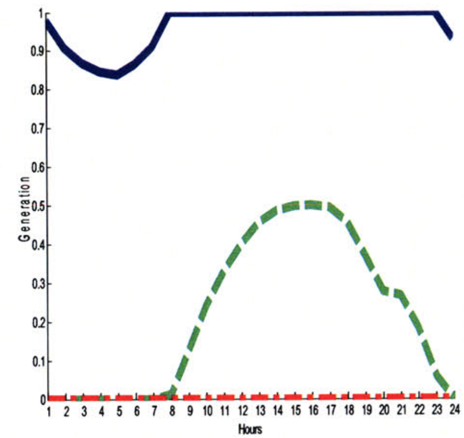
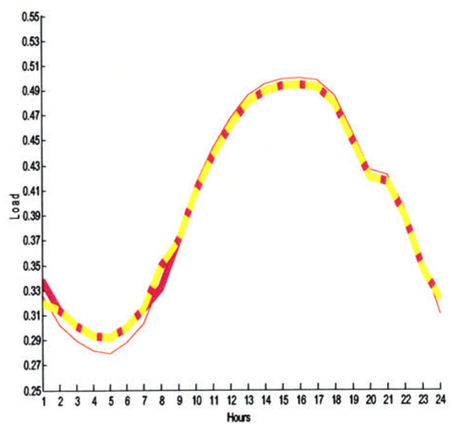
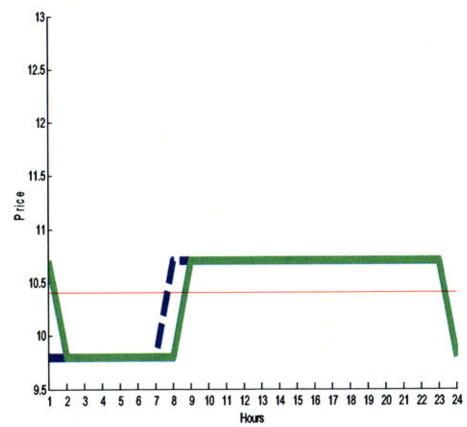
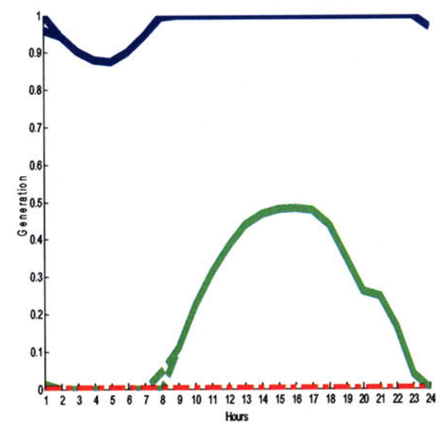
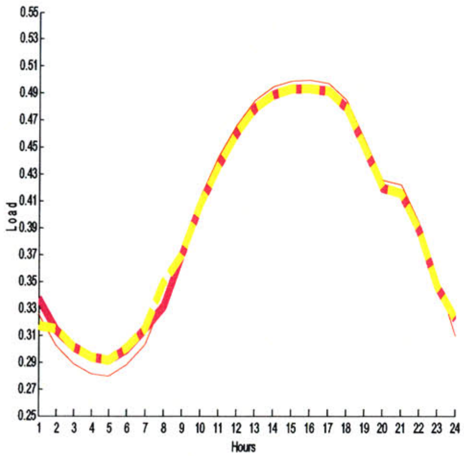
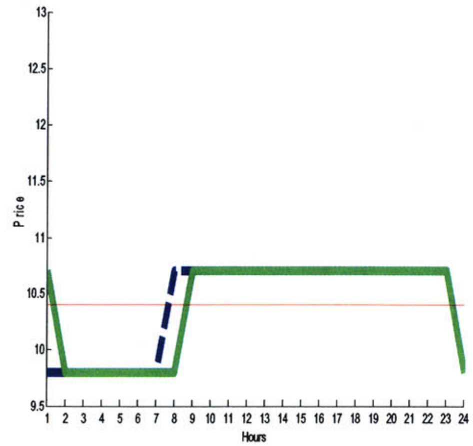
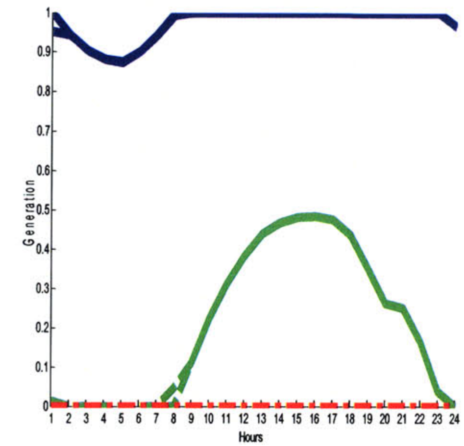
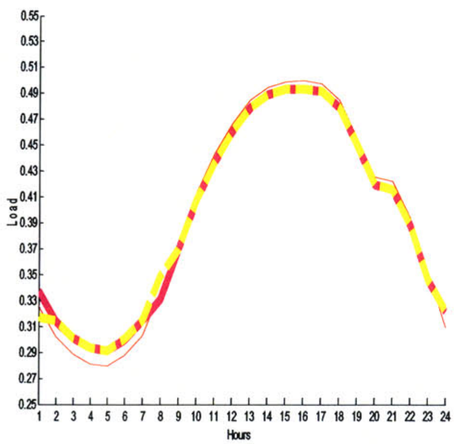
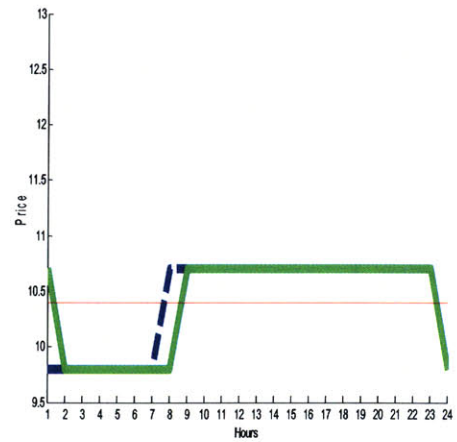
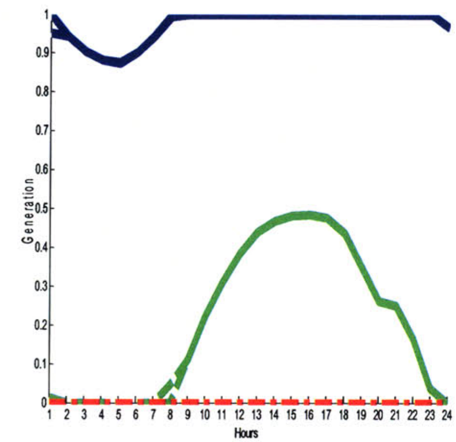
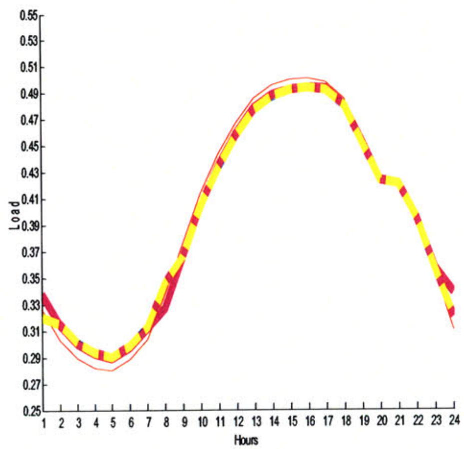
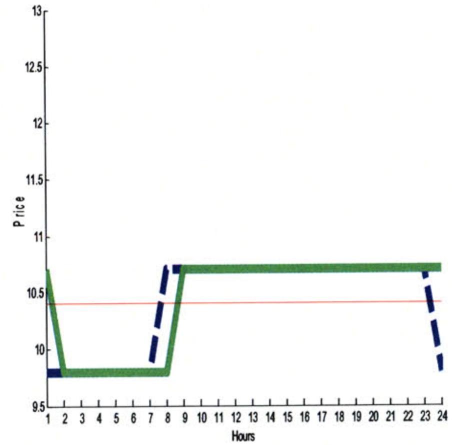
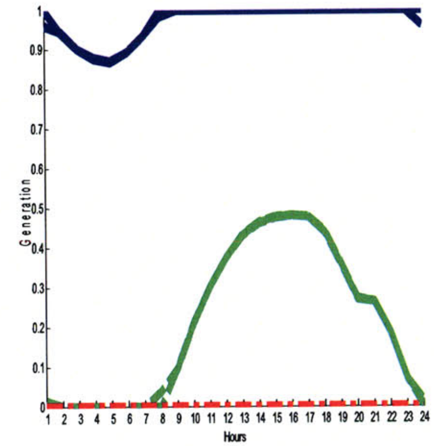
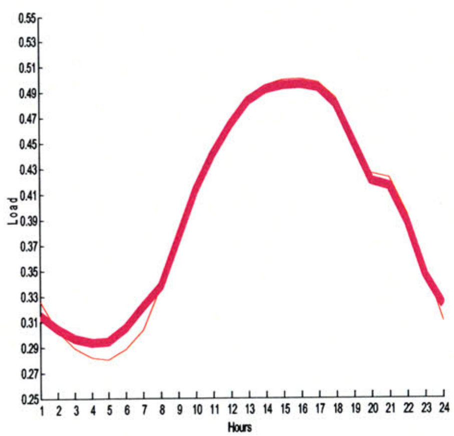
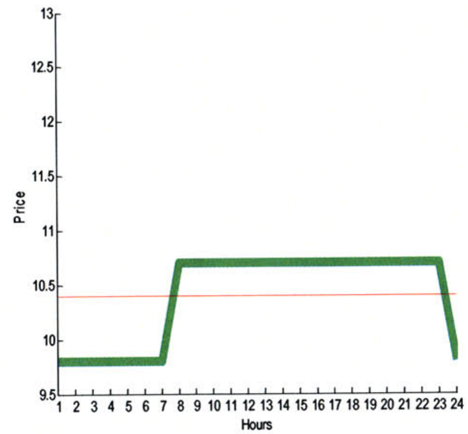
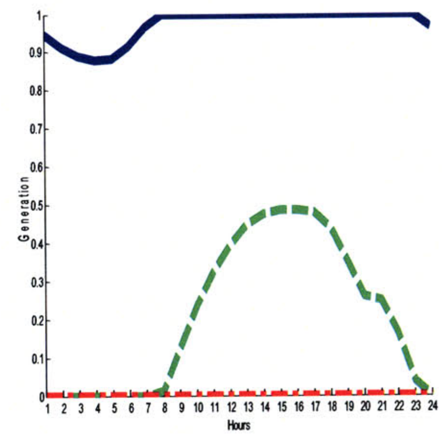
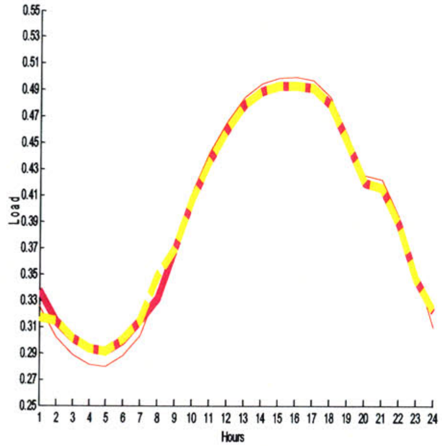
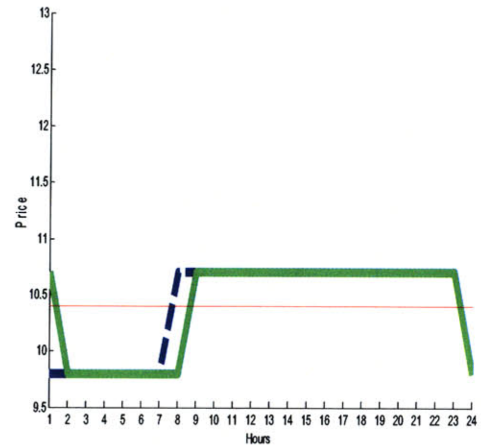
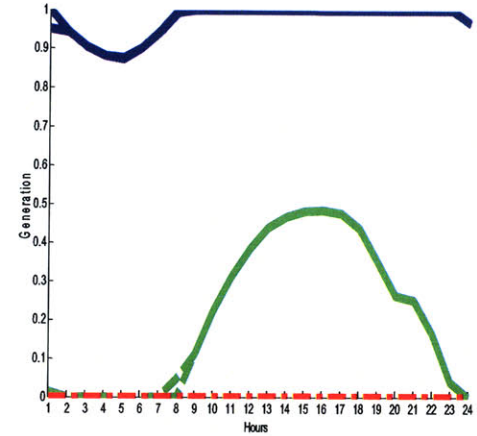
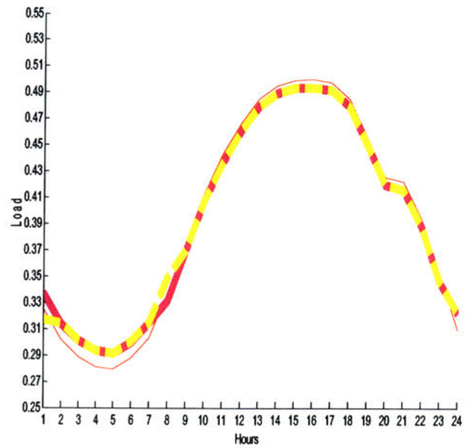
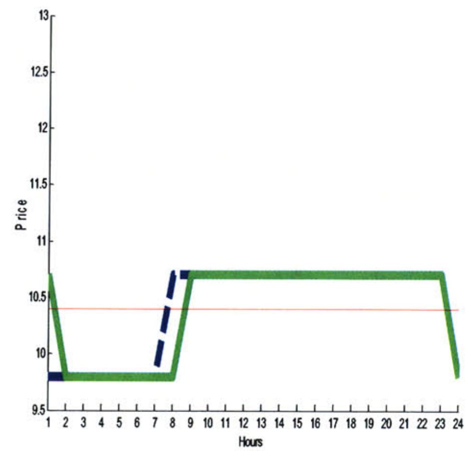
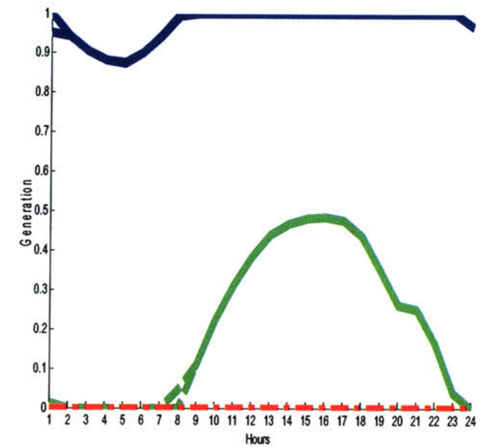
No.	Load Profile	Market Clearing Price	Generation Dispatch Schedule
9			
10			

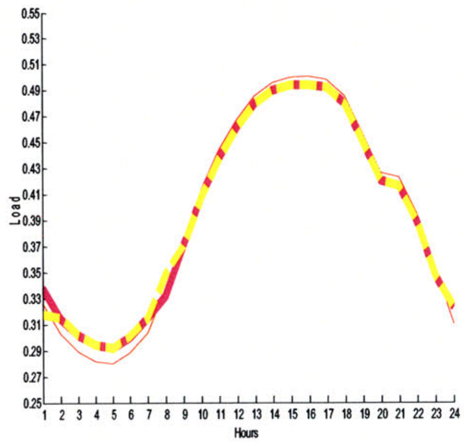
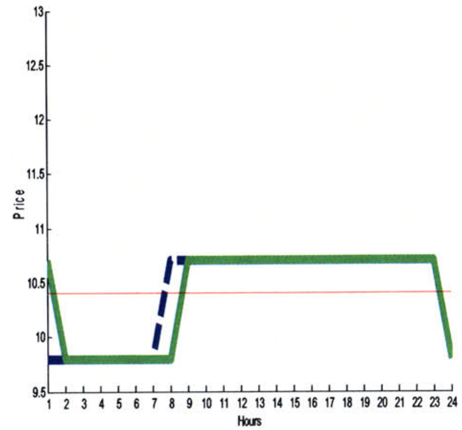
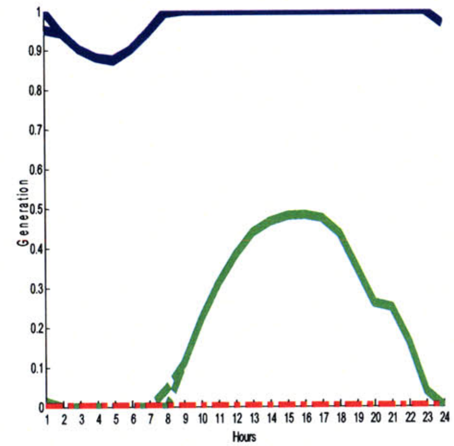
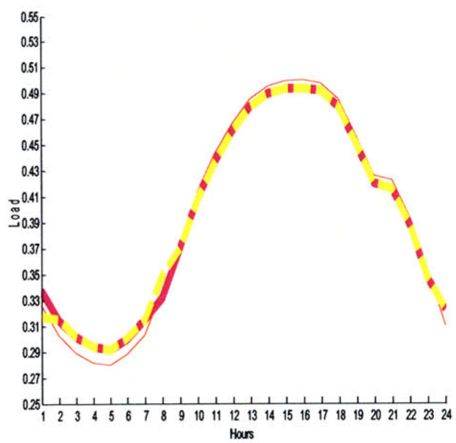
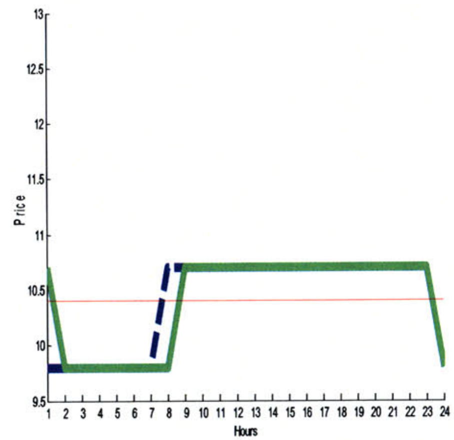
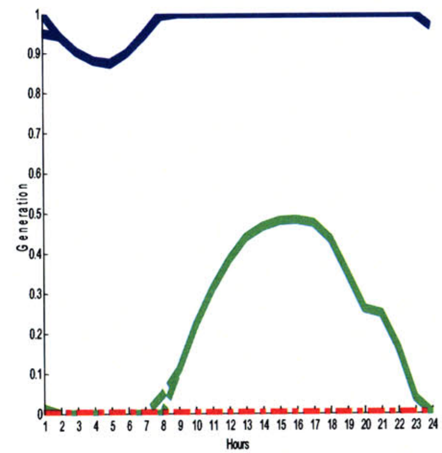
TABLE 4.2.4 Bidding results of the proposed bidding mechanism under nine end-user types (24 periods)

<table>
<tr><th>No.</th><th>Load Profile</th><th>Market Clearing Price</th><th>Generation Dispatch Schedule</th></tr>
<tr><td>1</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td></tr>
</table>

<table>
<tr><th>No.</th><th>Load Profile</th><th>Market Clearing Price</th><th>Generation Dispatch Schedule</th></tr>
<tr><td>3</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>4</td><td>
</td><td>
</td><td>
</td></tr>
</table>

No.	Load Profile	Market Clearing Price	Generation Dispatch Schedule
5			
6			

No.	Load Profile	Market Clearing Price	Generation Dispatch Schedule
7			
8			

<table>
<tr><th>No.</th><th>Load Profile</th><th>Market Clearing Price</th><th>Generation Dispatch Schedule</th></tr>
<tr><td>9</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>10</td><td>
</td><td>
</td><td>
</td></tr>
</table>

Case 8 is real-world end users. The market clearing prices are higher than the reference prices. Thus, the electric use is reduced to lower than the reference loads. The electric use first increases and then decreases. It is because shifting effects of the middle hours of the timeframe are stronger than those of the side hours. The shifting effects can be measured by equation (49).

Case 9 is distributed generation. Its PEM model is the same as curtailable load. However, two other constraints are added to model this end-user type. The first constraint is the distributed generation cannot operate as load. In other words, the load can only be reduced but not increased compared to the initial load value. The second constraint is that the distributed generation's capacity is 0.2. In other words, the load reduction cannot exceed 0.2. In this example, neither of the two constraints are touched. Therefore, the results are identical to those of case 2.

Case 10 is on-site storage. Its PEM model is the same as flexible end users. Moreover, the capacity constraint is added to model this end-user type. The capacity is set as 0.3 here. In this example, this constraint is unbounded, and thus the results are the same as those of case 7.

The numerical examples of flat references in Table 4.2.3 straightforwardly reflect the end-user types. All the results in this table are convergent. In practice, the end-user types can be described by more delicate models. For example, charge and discharge rate constraints can be added to the end-user type of on-site storage.

By analyzing all these numerical examples, we conclude that traditional bidding mechanism can cause great deviation in setting bidding results as market equilibriums. Figure 4.2.3 illustrate this statement. Figure 4.2.3 plots the Hour 2's bidding results of all the ten cases in Table 4.2.3. The inelastic demand bidding mechanism sets its bidding result at D, and the SHB set its bidding result at A. However, many end-user types have their actual market equilibrium settled apart from A and D. For this reason, low market efficiency, high RT market balancing cost and low reliability could be caused if applying the two traditional bidding mechanisms under these end-user types. Applying the proposed bidding mechanism may solve these potential problems.

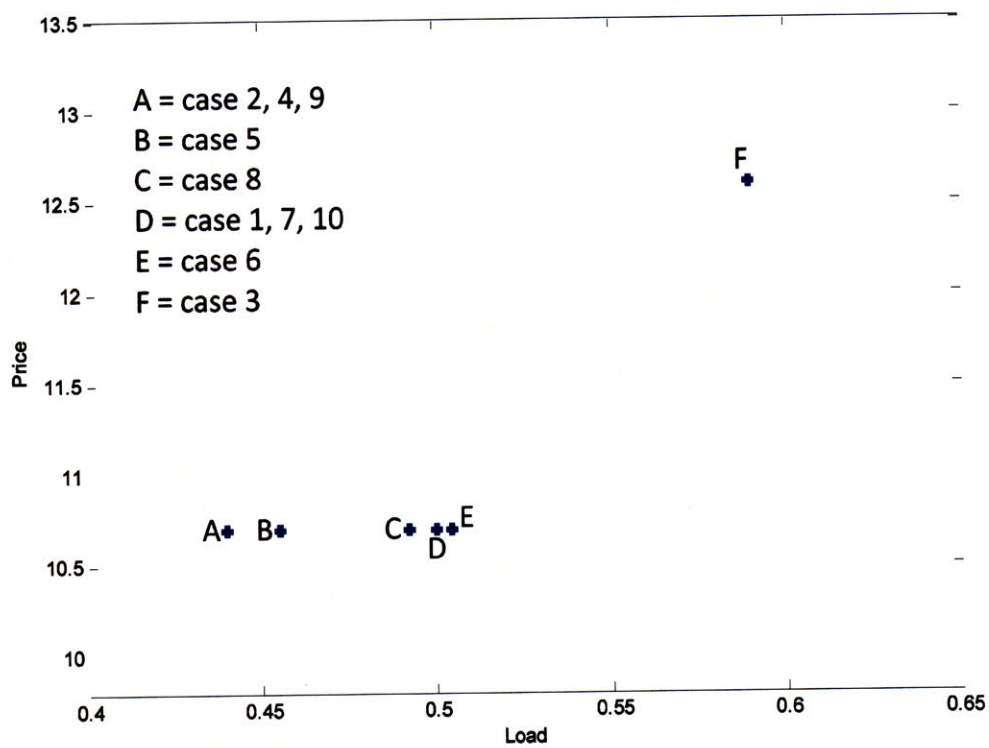


Fig. 4.2.3: Market equilibriums under the nice end-user types. The market equilibrium at Hour 2 under the proposed bidding mechanism, the inelastic demand bidding mechanism and the SHB.

The ten cases were also simulated with the 24-period data sets and are presented in Table 4.2.4. Numerical examples in Table 4.2.4 provide a rich view of the proposed bidding mechanism's performance. Analysis on these examples is similar to that of Table 4.2.3 and thus is not repeated here. Some of those numerical examples are non-convergent, and their results depict the final oscillation range. Section 4.5 will investigate into these non-convergent results in detail.

4.3 Example under Multiple End-User Types

Section 4.2 assumes the three retailers are of the same end-user types. In practice, the demand side is unlikely to have only a single end-user type, but it will include multiple end-user types. This section presents numerical examples of the proposed bidding mechanism under multiple end-user types. The simulation system contains three generation units and three retailers. The generation bids remain the same as data in Table 4.2.1.

This section examines the numerical examples in the DA market. The transaction period of the DA market is set as one hour. The three retailers have different electric use levels and shifting patterns. In other words, their bids are distinct in reference points and PEMs. Two data sets are used as the retailers' bids. The first data set is of five-period timeframe. The reference prices and loads are flat in time, shown in figure 4.3.1.

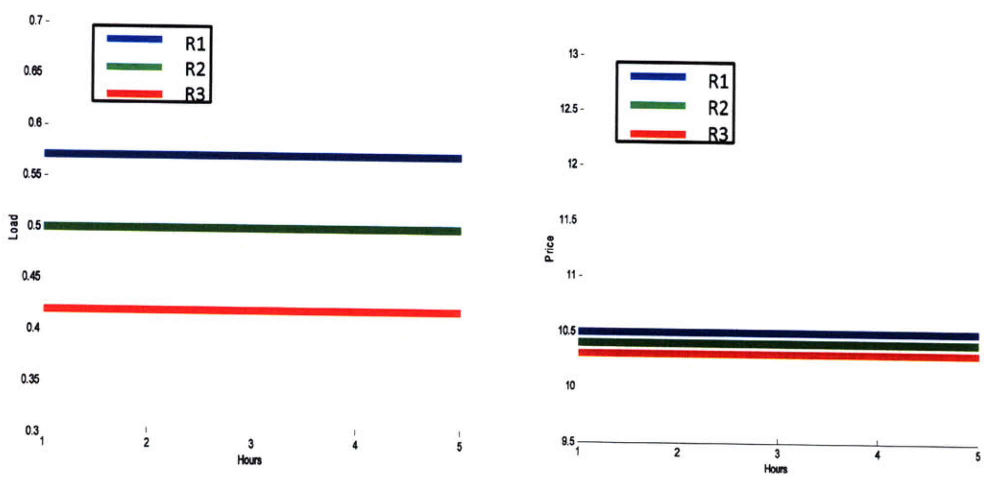


Fig. 4.3.1 Reference load and price of five-period retailers' bids

The second data set is of 24-period timeframe. The reference prices and loads are derived from the electric use of New York City, Long Island and Dunwod New York, in August 9, 2008. The data source is introduced in Section 4.1. Figure 4.2.2 depicts the reference points.

The transaction timeframe of the DA market equals to the timeframe of the data set applied. The initial load values are set the same as the reference loads. In addition, we set maximum loads to 1.0 and minimum loads to 0.0.

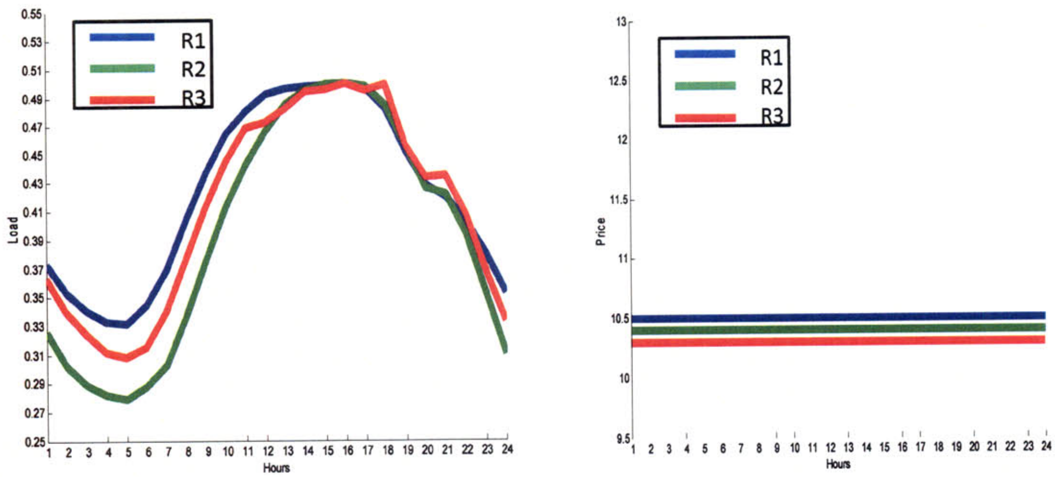


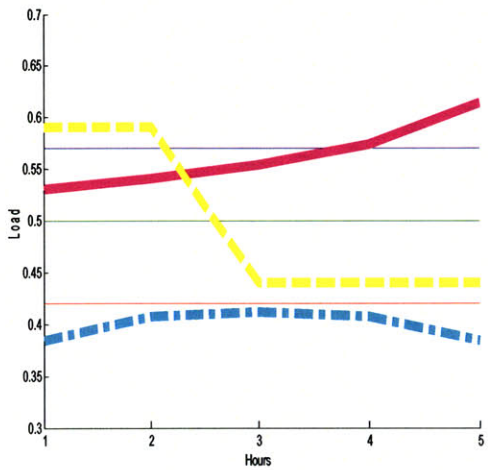
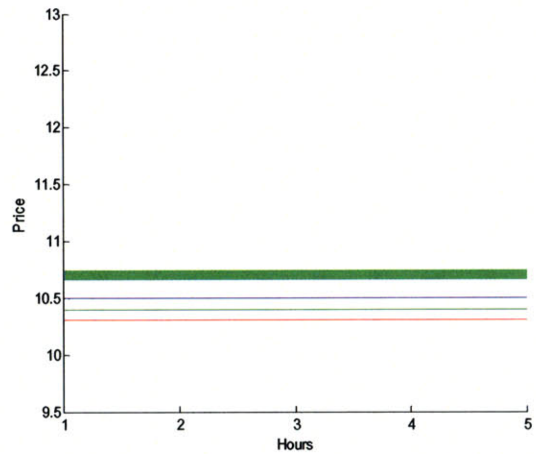
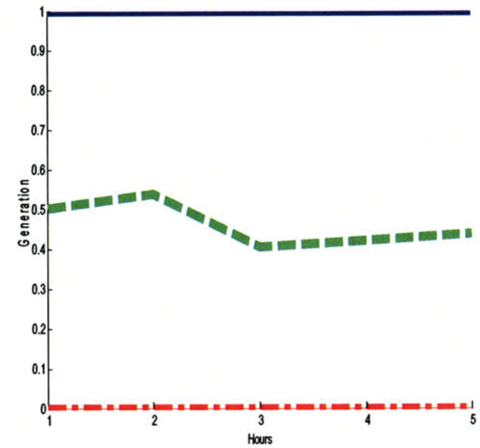
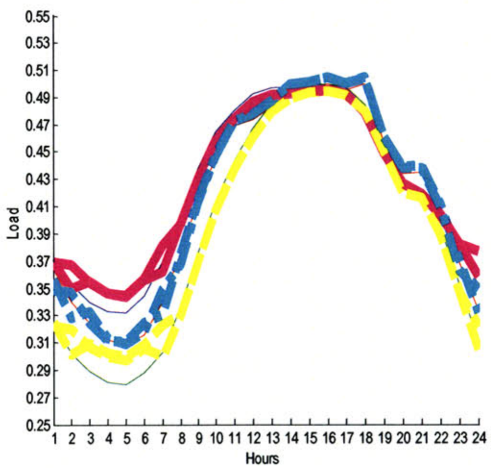
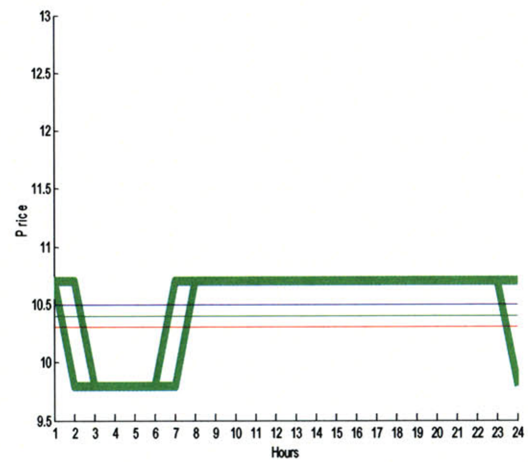
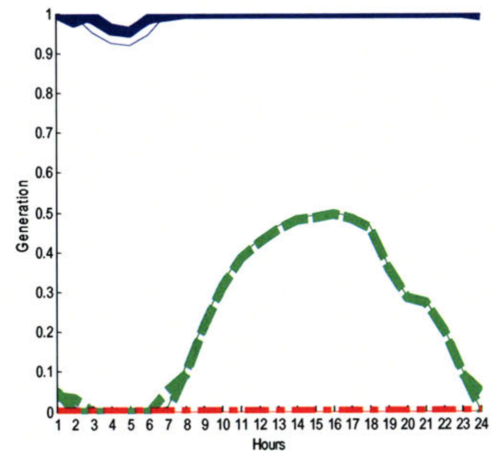
Fig. 4.3.2 Reference load and price of 24-period retailers' bids. Retailer 1, R1, uses reference points derived from data of N.Y.C.; Retailer 2, R2, uses reference points derived from data of Long Island; Retailer 3, R3, uses reference points derived from data of Dunwod.

The end-user types of the three retailers are forward shifting end users, early end users and real-world end users. These end-user types are modeled by PEMs as described in Table 4.2.2. The bidding results of the numerical examples under these end-user types are presented in Table 4.3.1. Reference points are plotted together with the bidding results. Case 1 shows the bidding results under the three end-user types with the five-period data set; Case 2 shows the bidding results with the 24-period data set.

In Case 1, the early end users and forward shifting end users shift according to their pattern, since the market clearing prices are higher than the reference prices. For the same reason, the real-world end users reduce their electric use and shift loads to the middle hour of the timeframe. The generation dispatch schedule shows that the total loads represent the mixed shifting behavior of all the three end-user types. We can also predict that the total load will represent a certain end-user type if this type of end users occupies a large electric use portion.

Case 2 provides a rich view of the bidding mechanism's performance under the three end-user types. This example is not convergent, and the results depict the final oscillation range. Section 4.5 will investigate into these non-convergent results in detail.

TABLE 4.3.1 Bidding results of the proposed bidding mechanism under multiple end-user types

No.	Load Profile	Market Clearing Price	Generation Dispatching Schedule
1			
2			

4.4 Examples under Systems with Contingencies

Section 4.2 and Section 4.3 examine the performance of the proposed bidding mechanism under ordinary system state. This section gives numerical examples under contingent system status. The simulation system contains three generation units and three retailers.

This section examines the numerical examples in the DA market. The transaction period of the DA market is set as one hour. The three retailers are considered identical in their reference points and end-user types. Two data sets are used as the retailers' bids. The first data set is of five-period timeframe. The reference prices and loads are flat in time, shown in figure 4.2.1. The second data set is of 24-period timeframe, representing practical daily consumption pattern. Figure 4.2.2 depicts the reference points. The source of the two data sets is introduced in Section 4.1.

The end-user type of the three retailers is real-world end users, which is modeled by the PEM described in Table 4.2.2. The transaction timeframe of the DA market equals to the timeframe of the data set applied. The initial load values are set the same as the reference loads. In addition, we set maximum loads to 1.0 and minimum loads to 0.0.

Given the generation bids in Table 4.2.1, we consider two types of major generation-side contingencies. The first type of contingency is sudden change of generation costs. This type of contingency happens when fuel cost changes, generator maintenance takes place and suppliers game in the bidding process. The second type of contingency is sudden loss of generation. This type of contingencies happens when generators break down, run out of fuel and are lack of renewable energies, for example, wind turbine cannot generate power when wind stops blowing. Since contingencies usually only happen to part of generation units in a power system, we assume the two types of contingencies happen on the generation unit G2. Therefore, G2's capacity and marginal cost are no longer time-invariant. Figure 4.4.1 to figure 4.4.4 plot the changes of G2's capacity and marginal cost due to these types of contingencies. In addition, as mentioned in Section 4.1, because network constraints are not considered in the numerical example settings for the data analyzing purpose, the transmission contingencies are not examined in this section.

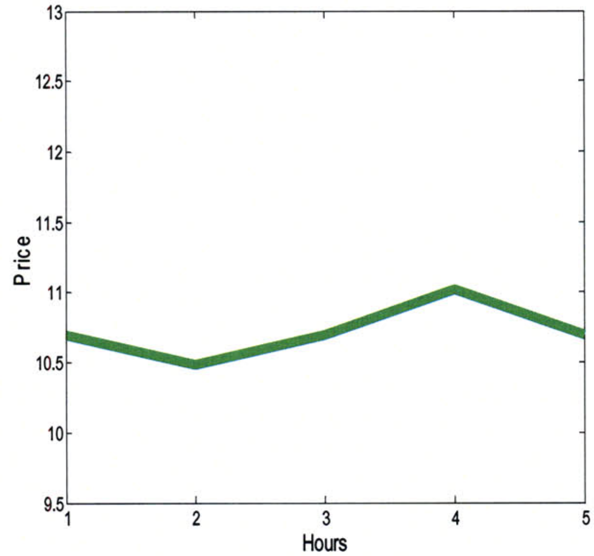


Fig. 4.3.1 G2's generation cost. G2's marginal cost is bid as 10.7. This marginal cost is forecasted to suddenly decrease at Hour 2 to 10.49 and increase at Hour 2 to 11.02.

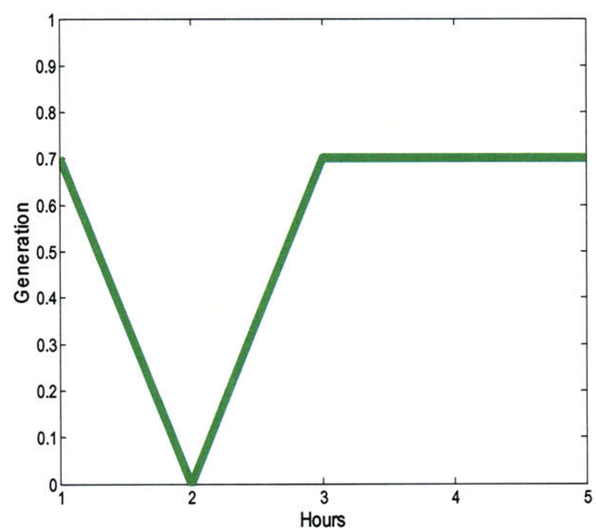


Fig. 4.3.2 G2's generation capacity. G2's capacity is bid as 0.7. This capacity is forecasted to be lost (i.e. drop to 0.0) at Hour 2.

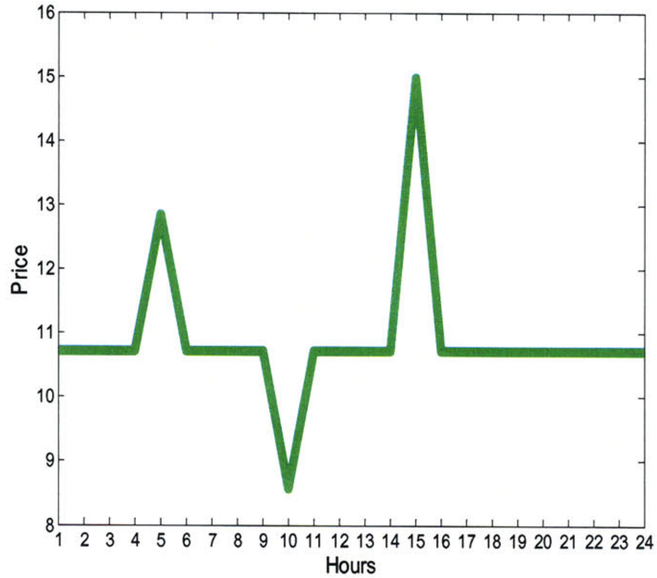


Fig. 4.3.3 G2's generation cost. G2's marginal cost is bid as 10.7. This marginal cost is forecasted to suddenly increase at Hour 5 and Hour 10 to 12.84 and 14.98, and decrease at Hour 10 to 8.56.

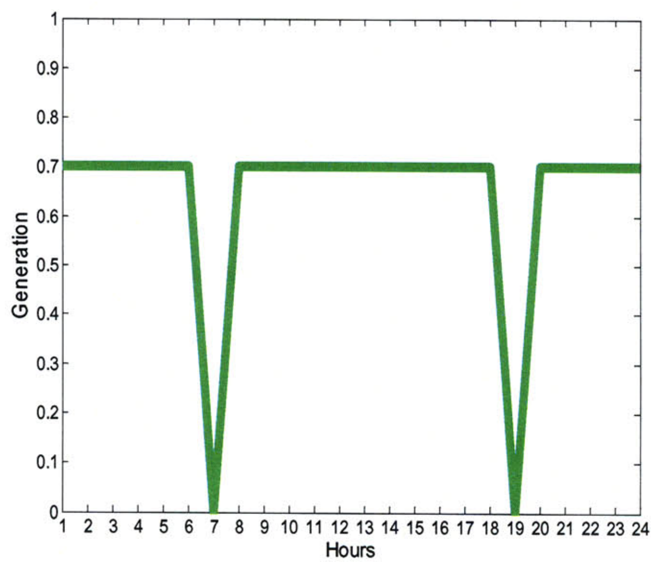


Fig. 4.3.2 G2's generation capacity. G2's capacity is bid as 0.7. This capacity is forecasted to be lost (i.e. drop to 0.0) at Hour 7 and Hour 19.

Table 4.4.1 presents the five period bidding results of the proposed bidding mechanisms under these two contingencies. Table 4.4.2 and Table 4.4.3 presents the same numerical examples under the inelastic demand bidding mechanism and the SHB.

Case 1 shows the bidding results when the system has G2's cost suddenly changed at Hour 2 and Hour 4. Under all the three bidding mechanisms, G2 is the marginal unit at all the hours. Therefore, the market clearing prices of the three bidding mechanisms are equal to G2's marginal cost. The bidding results of the three bidding mechanisms differ in their load profiles and generation dispatch schedule.

As shown in Table 4.4.2, under the inelastic demand bidding mechanism, the load profile is constant under the time-varying prices. End users' electric use is the same under the peak price as under the off-peak price, which increases end-user consumption cost. Moreover, according to equation (24), this bidding result reduces the social welfare value. As shown in Table 4.4.3, under the SHB, end users reduce their electric use, since the market clearing prices are higher than the reference prices at all the hours. End users increase their electric use at low prices and reduce their electric use at high prices. However, inter-temporal load shifting is ignored in this bidding mechanism. For this reason, at all the hours the end-user electric use is reduced, which in turn leads to the reduction of end-user utility and social welfare according to equation (18) and equation (24). The bidding results of these two bidding mechanisms deviate from the actual market equilibrium.

The bidding result of the proposed bidding mechanism gives the optimal solution under the given end-user type, as shown in Table 4.4.1. Since the market clearing prices are higher than the reference prices, the total end-user electric use is reduced. End users increase their electric use at low prices and reduce their electric use at high prices. Meanwhile, the end-user electric use is increased above the reference load at the low-priced hour due to the load shifting from other hours. In this bidding result, the end-user electric use is controlled by time-varying prices to reduce total generation cost. Moreover, sufficient end user electric use is guaranteed by inter-temporal load shifting to ensure the end-user utility level.

TABLE 4.4.1 Bidding results of the proposed bidding mechanism under system contingencies (5-period)

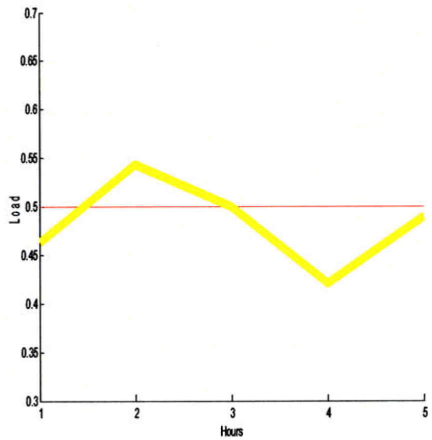
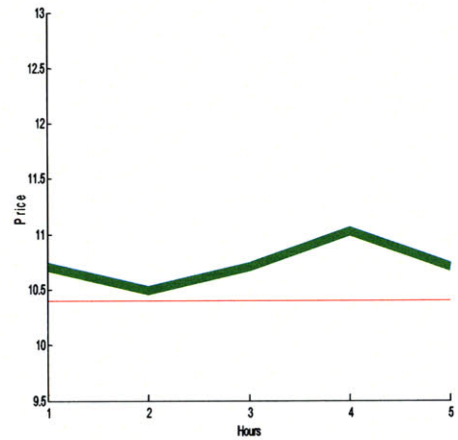
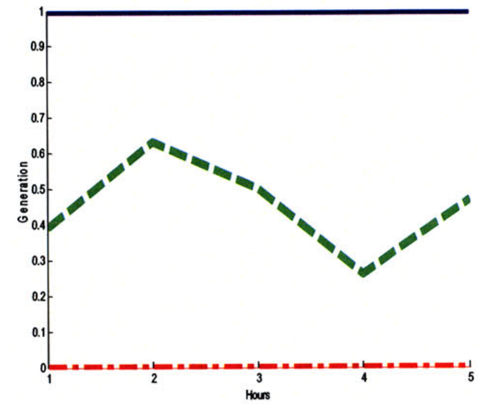
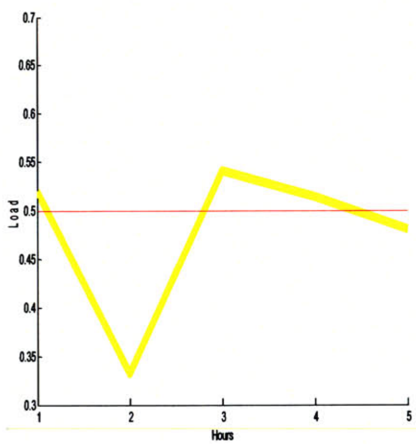
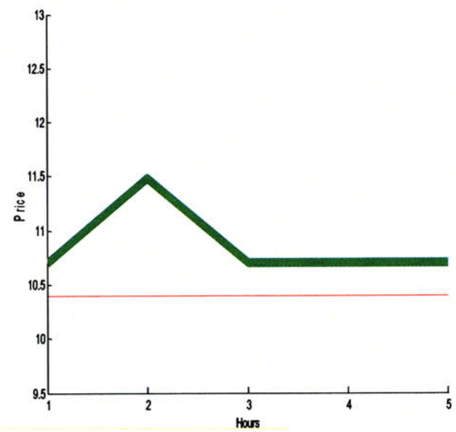
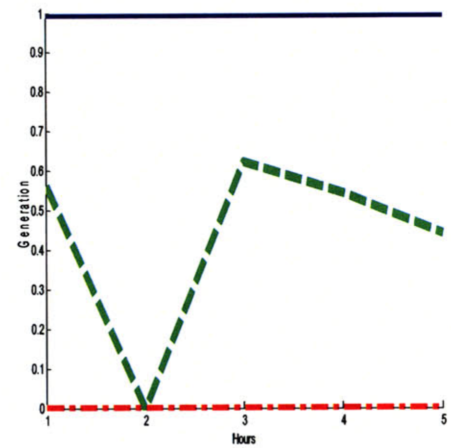
No.	Load Profile	Market Clearing Price	Generation Dispatch Schedule
1			
2			

TABLE 4.4.2 Bidding results of the demand inelastic bidding mechanism under system contingencies (5-period)

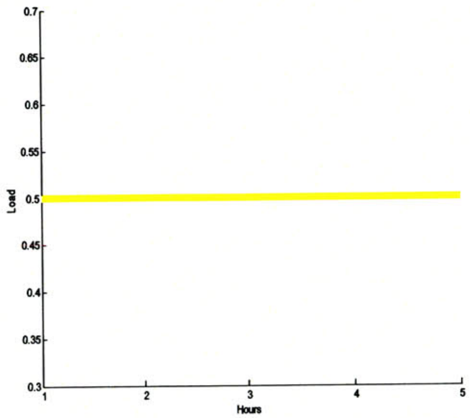
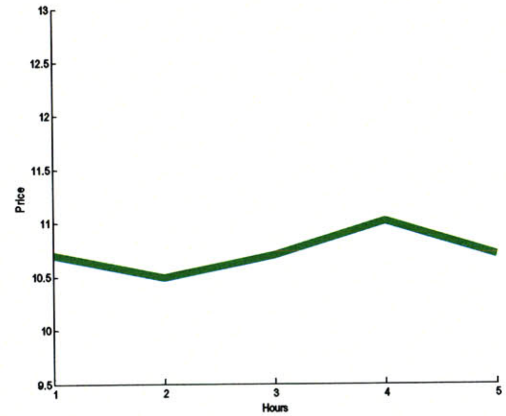
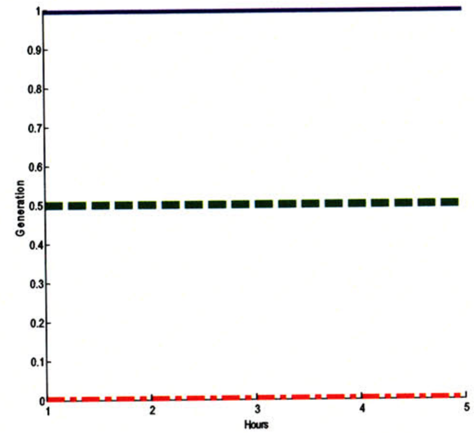
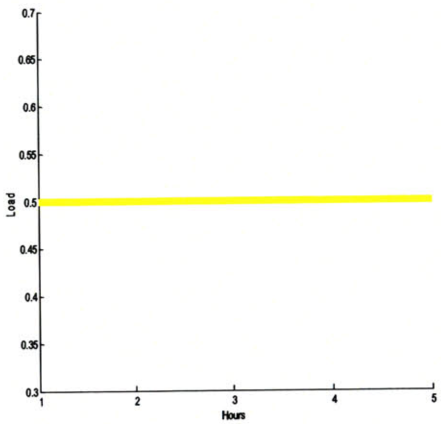
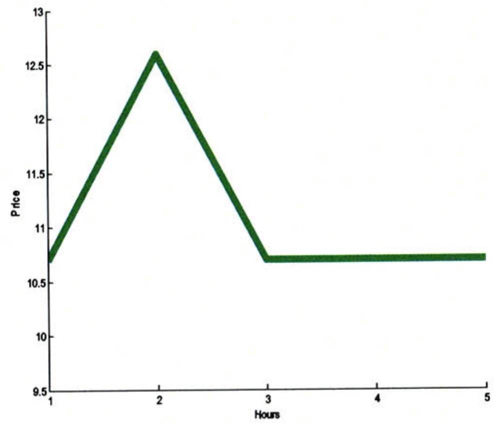
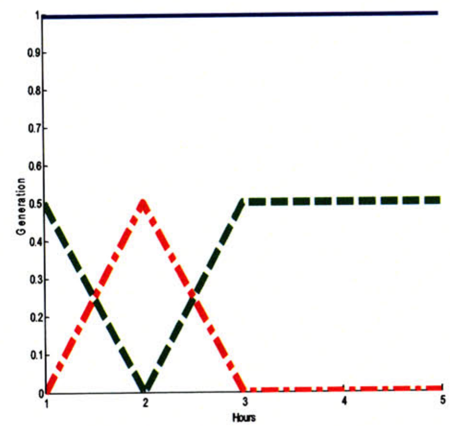
No.	Load Profile	Market Clearing Price	Generation Dispatch Schedule
1			
2			

TABLE 4.4.6 Bidding results of the Single Hourly Bidding (SHB) under system contingencies (5-period)

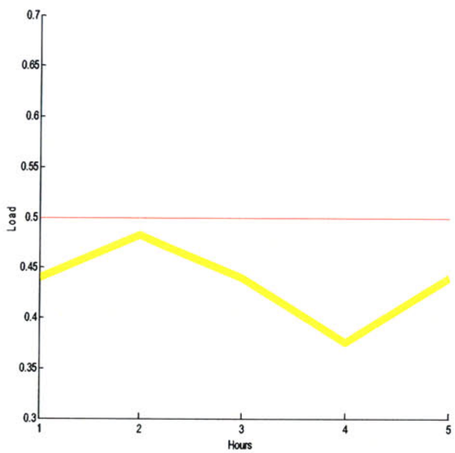
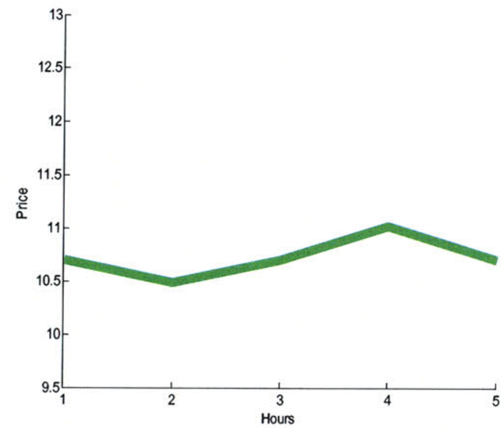
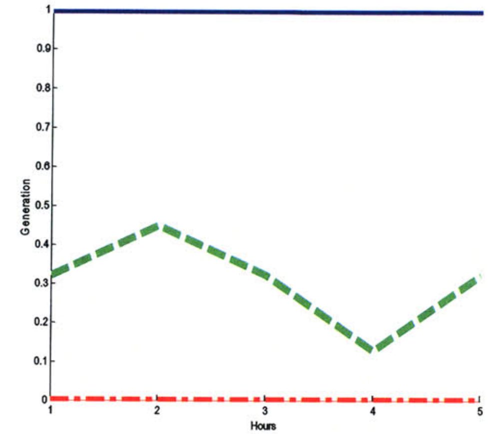
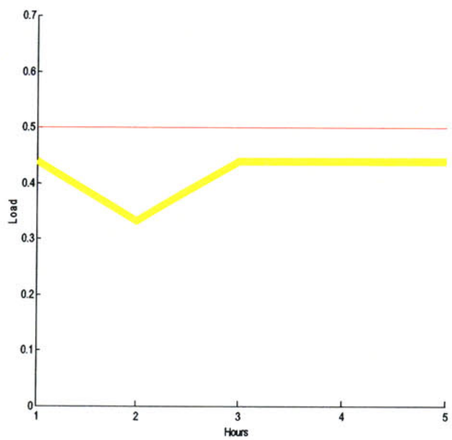
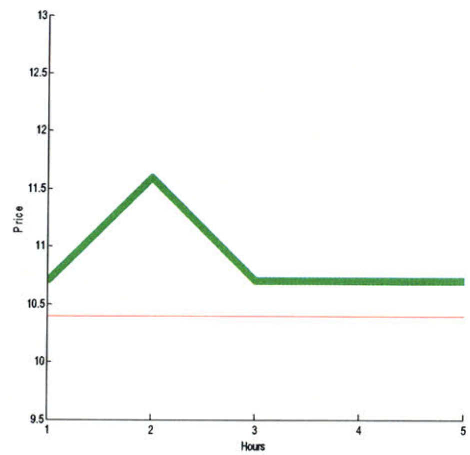
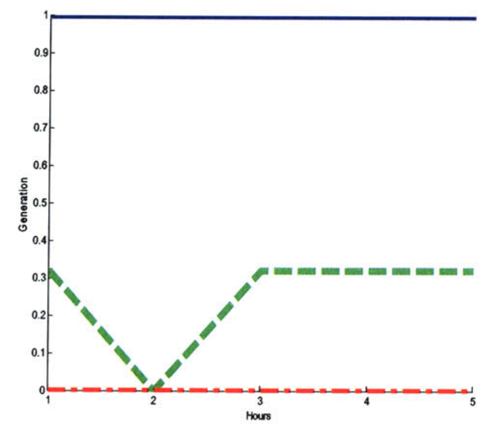
No.	Load Profile	Market Clearing Price	Generation Dispatch Schedule
1			
2			

TABLE 4.4.4 Bidding results of the proposed bidding mechanism under system contingencies (24-period)

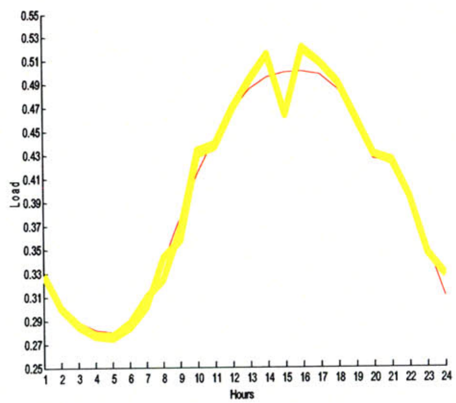
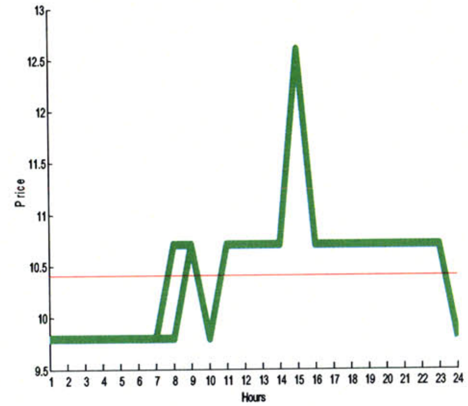
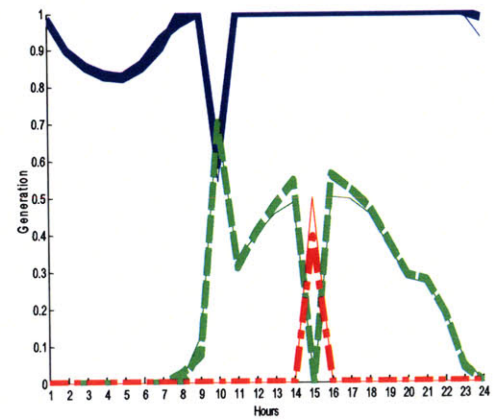
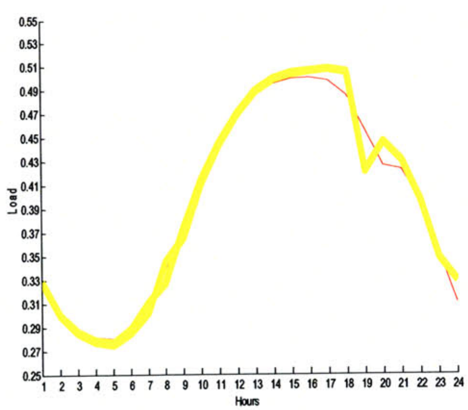
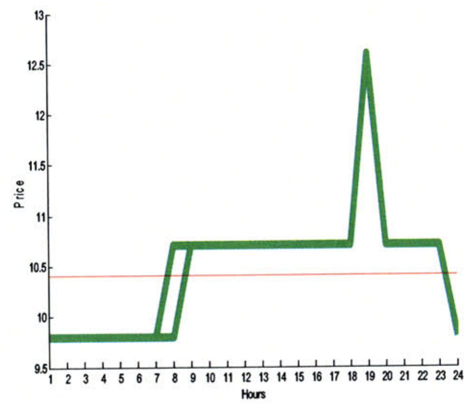
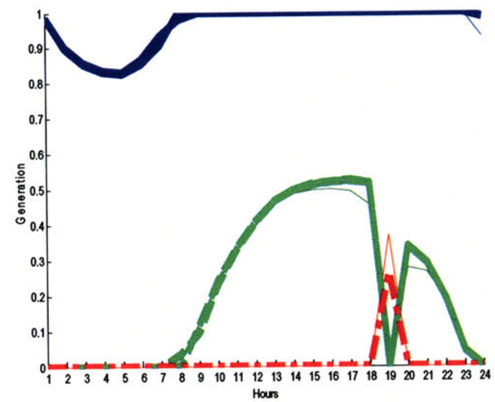
No.	Load Profile	Market Clearing Price	Generation Dispatch Schedule
1			
2			

TABLE 4.4.5 Bidding results of the inelastic demand bidding mechanism under system contingencies (24-period)

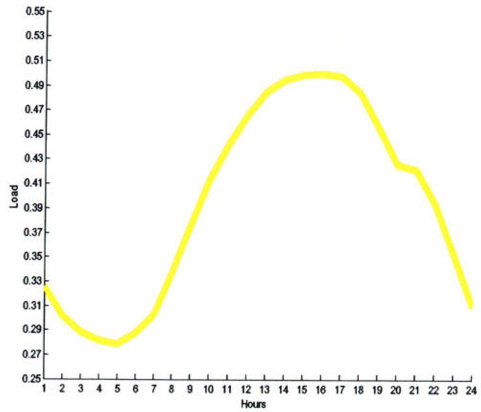
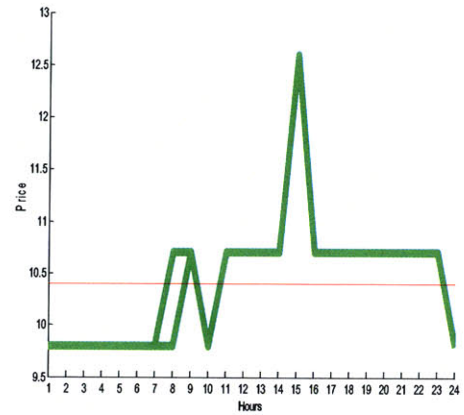
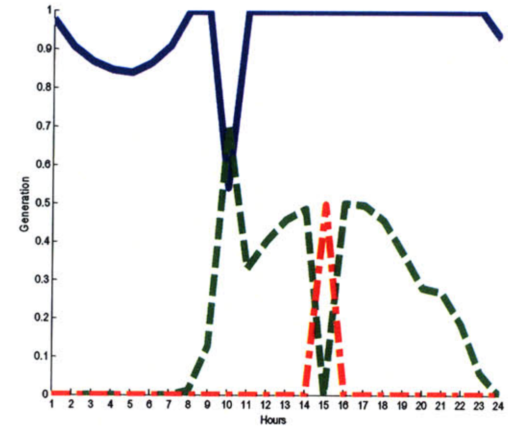
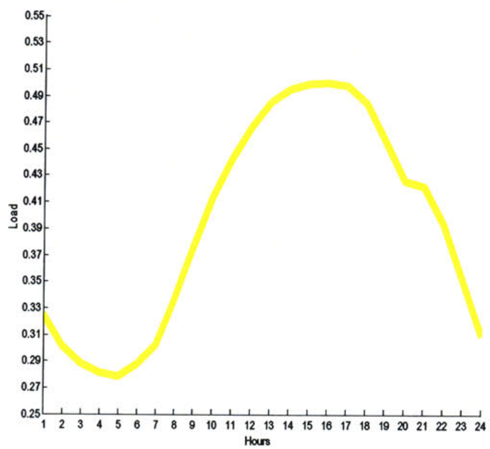
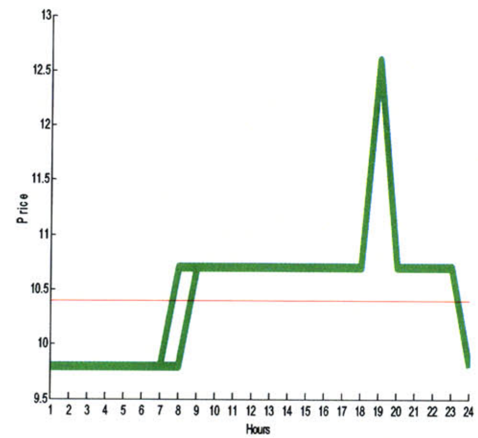
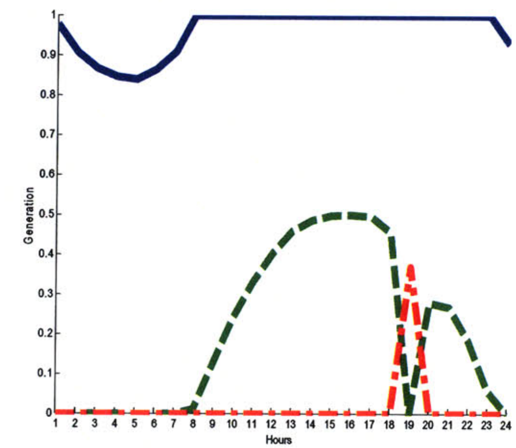
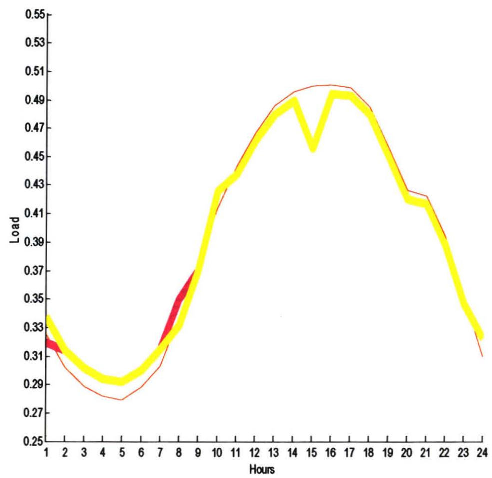
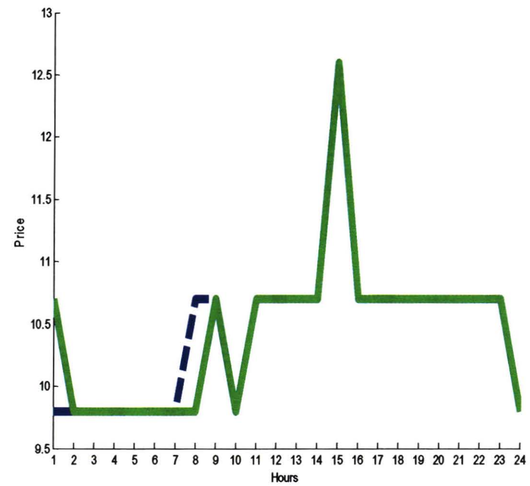
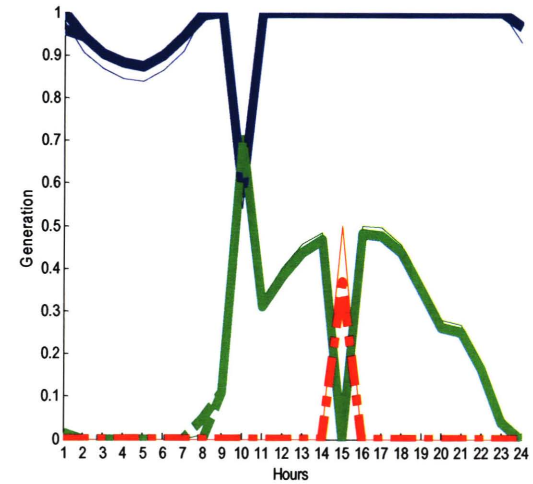
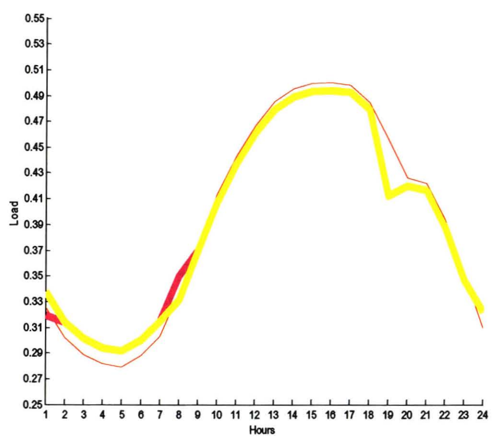
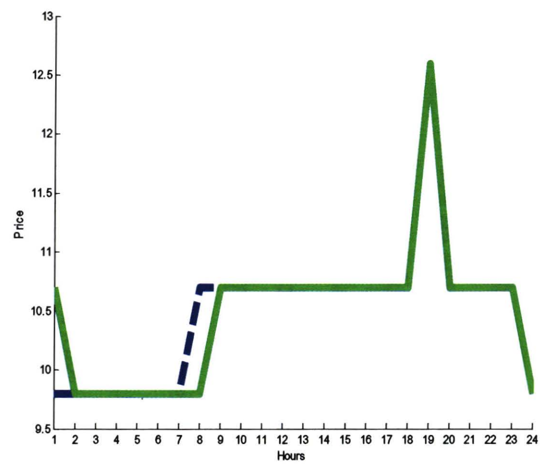
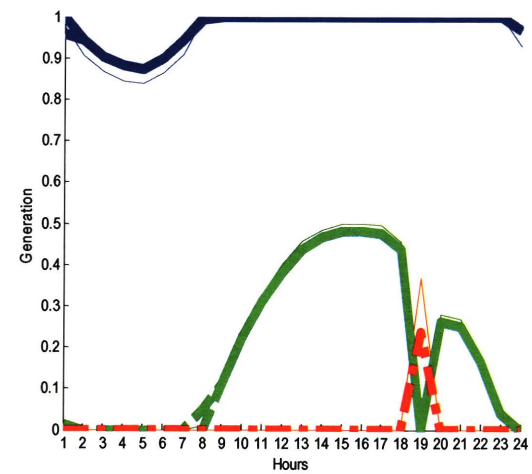
No.	Load Profile	Market Clearing Price	Generation Dispatch Schedule
1			
2			

TABLE 4.4.6 Bidding results of the Single Hourly Bidding (SHB) under system contingencies (24-period)

No.	Load Profile	Market Clearing Price	Generation Dispatch Schedule
1			
2			

Case 2 shows the bidding results when the system suddenly loses G2 at Hour 2. The market clearing prices of all the three bidding mechanisms increase at Hour 2 due to this contingency.

As shown in Table 4.4.2, under the demand inelastic bidding mechanism, the end users have the same electric use at Hour 2 as at the other hours. In order to satisfy this demand, the most expensive generation unit, G3, is dispatched at Hour 2, which pushes the market clearing prices to a higher level. This bidding result will increase the end-user consumption cost, increase the total generation cost and thus reduce the social welfare according to equation (24). As shown in Table 4.4.3, under the SHB, the end users reduce their electric use at all the hours since the market clearing prices are higher than the reference price. Moreover, they consume the least electricity at Hour 2 when the price is peaking. This load profile makes G2 as the marginal units at all the hours, which saves the total generation cost compared to the bidding results of the inelastic demand bidding mechanism. However, the inter-temporal load shifting effects are ignored here, and thus the end-user utility is reduced according to equation (18).

The bidding result of the proposed bidding mechanism is presented in Table 4.4.1. The end users reduce their electric use since the market clearing prices are higher than the reference prices. They consume the least electricity at the peak hour and shift the electric use to the other hours. To satisfy this demand, G2 is the marginal unit at all the hours. In this bidding result, the end-user electric use is controlled in the way that saves the total generation cost, maintain the end-user utility above a certain level and thus gives a higher social welfare. Moreover, the market clear prices of Hour 2 under the three bidding mechanisms are 11.48 (Table 4.4.1), 12.6 (Table 4.4.2) and 11.6 (Table 4.4.3). It shows that under contingencies, the proposed bidding mechanism reduce the price spike most compared to the two traditional bidding mechanisms. Section 4.6 will analyze this effect in HA market when final payments are settled. Notice that the market clearing prices as 11.48 and 11.6 are not any generation units' marginal cost. These market clearing price are obtained due to demand clears the market at Hour 2. Section 4.5 will investigate into this problem in detail.

Table 4.4.4 to Table 4.4.6 provide 24-period bidding results of the proposed bidding mechanism together with those of the two traditional bidding mechanisms under the two contingencies. The analysis of these examples is identical to that of the 5-period results, and thus is not repeated here. Some of the examples are non-convergent, and the results give their oscillation range. Section 4.5 will show how to derive the market equilibriums of these results.

4.5 Examples under the System with Renewable Energy in DA and HA markets

Section 4.2 to Section 4.5 examines the performance of the proposed bidding mechanism in the DA market. This section shows how the proposed bidding mechanism in the HA market. The 24-period data set is used as the retailers' bids. Figure 4.2.2 depicts the reference points. The source of the two data sets is introduced in Section 4.1.

A wind turbine is introduced to the generation side in the simulation system, which contains three generation units and three identical retailers. The wind turbine is modeled by G2, and thus G2's marginal cost is set as 0.0. The wind capacity is modeled with Weibull distribution function:

$$f(x;\lambda,k) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^{k}} \qquad x \geq 0,$$

where $k = 7$ and $\lambda = 1$.

The operation procedure of the HA market is similar to that of the RT market, which is introduced in Section 3.2. In this section, the HA market's timeframe equals to 24, representing 24 hours in a day. The bidding process is held hourly, and thus the market are cleared 24 times when a whole run of the HA market is finished. In every hourly bidding process, no new bids are required from the suppliers or the retailers. However, both the generation bids and retailers' bids are updated.

Before every hourly transaction, generation bids are updated with the wind capacity's forecast for all the future hours in the timeframe. The updated wind capacity is used as G2's capacity of the generation bids at that hour. The "wind capacity" column of Table 4.5.1 shows the wind capacity's forecast in every hour. The rest of the generation bids remain the same as in Table 4.2.1.

The retailers' bids are updated hourly as well. Since the hourly biddings only transact the electric use in the further hours in the timeframe, the PEM is updated as the right bottom square sub-matrix of the original PEM. For example, if the HA market takes place in Hour 12, then the PEM at that hour is the sub-matrix containing entries of column 12 to 24 and row 12 to 24 from the original 24×24 matrix. Figure 4.5.1 illustrates this idea.

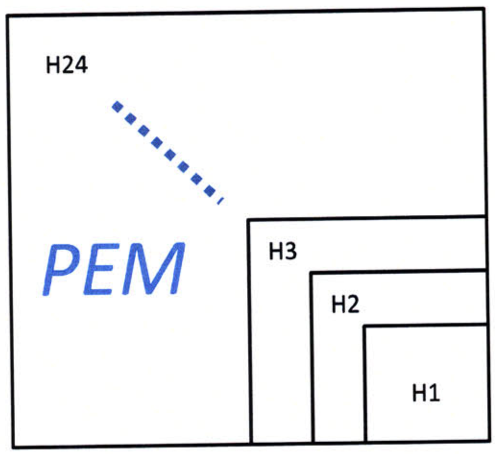
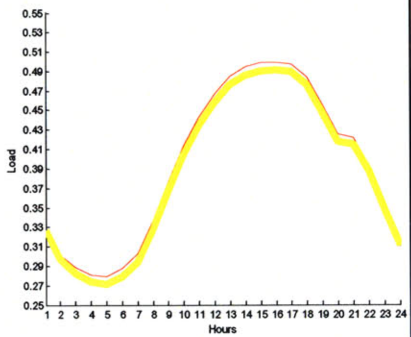
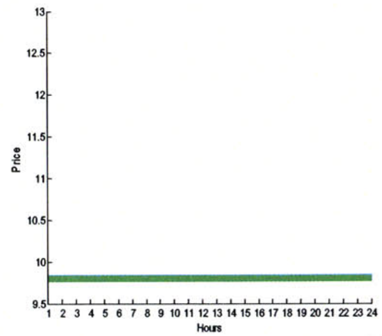
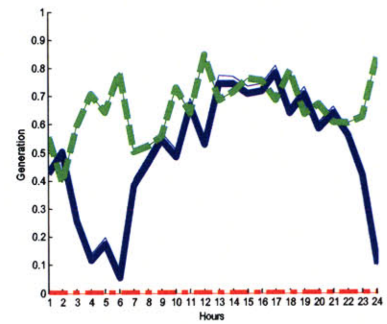
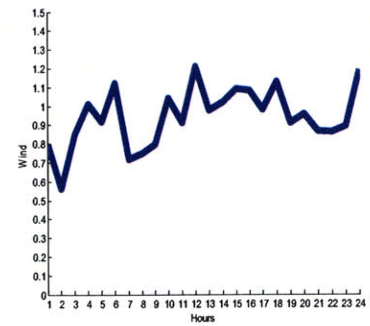
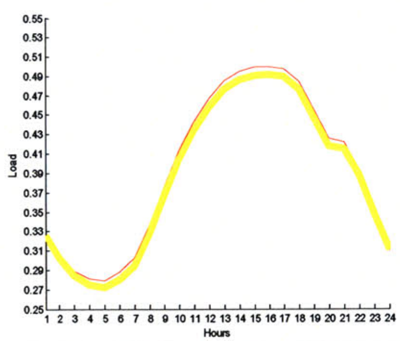
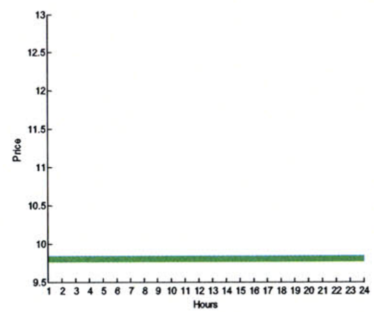
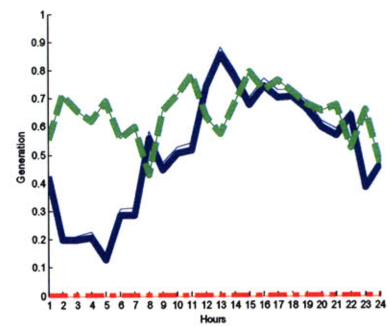
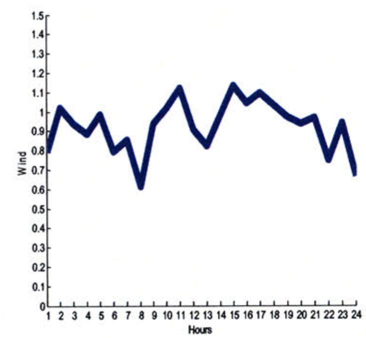
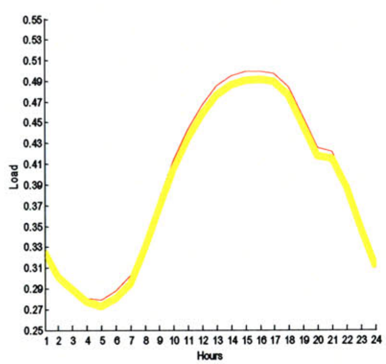
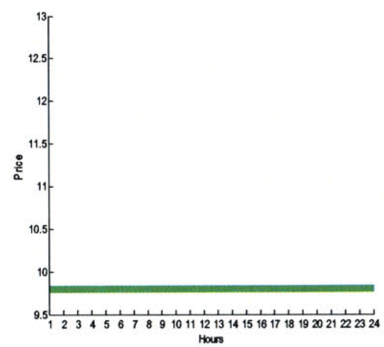
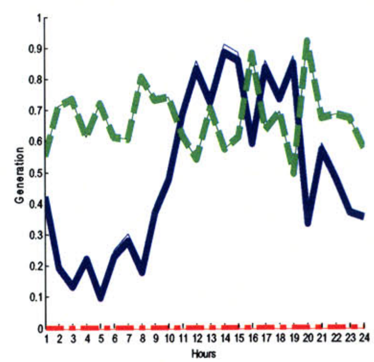
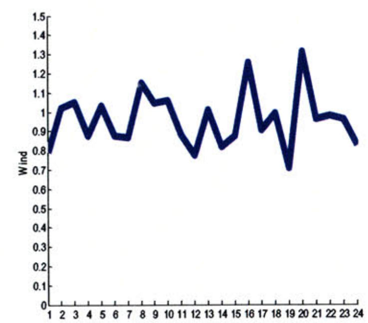


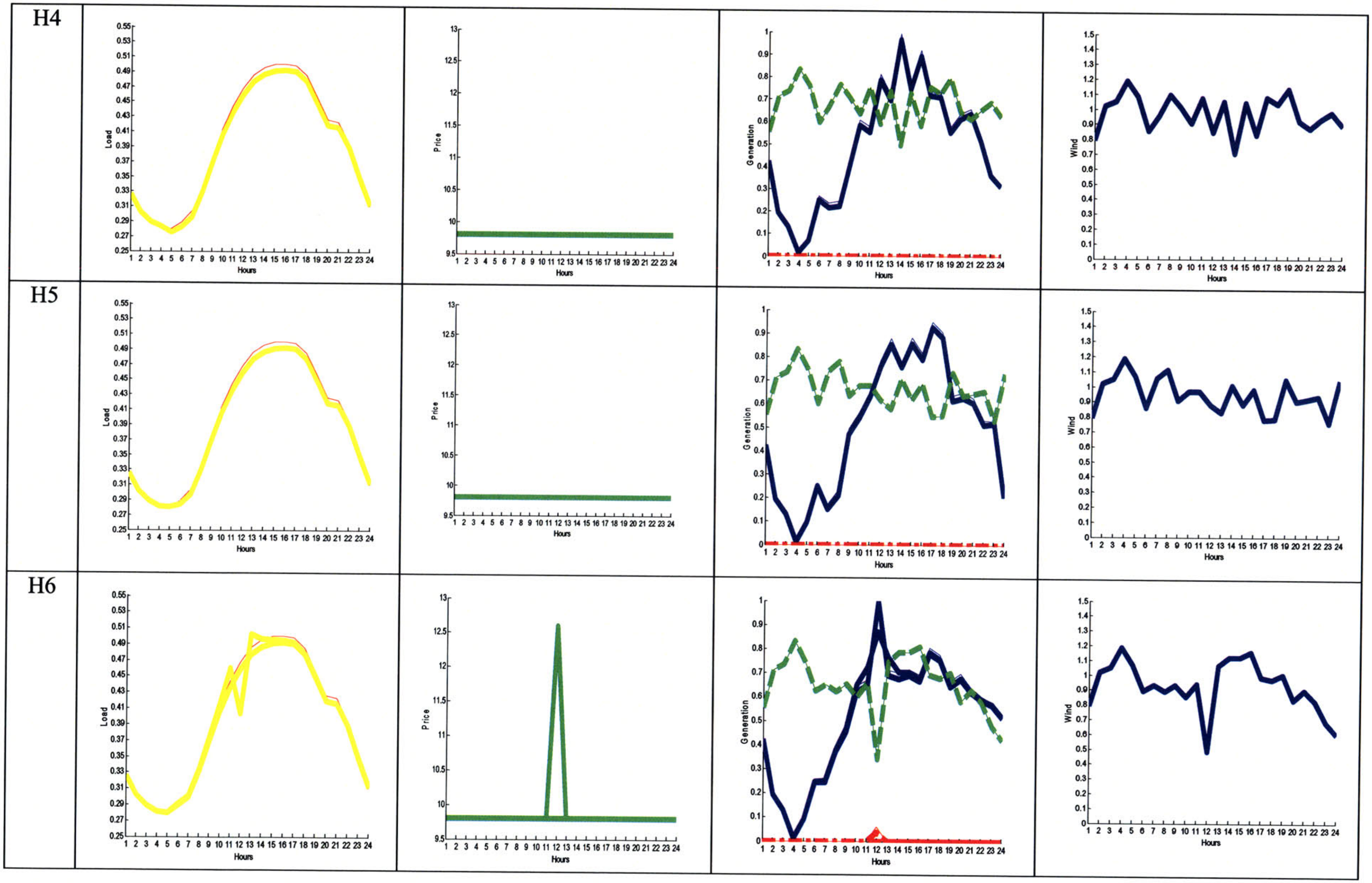
Fig. 4.5.1 Updated PEM for hourly bidding process in the HA market.

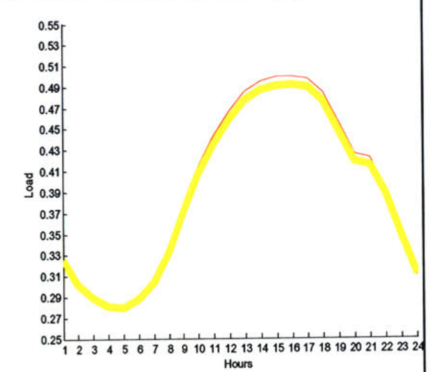
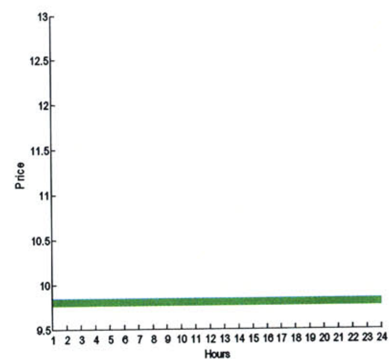
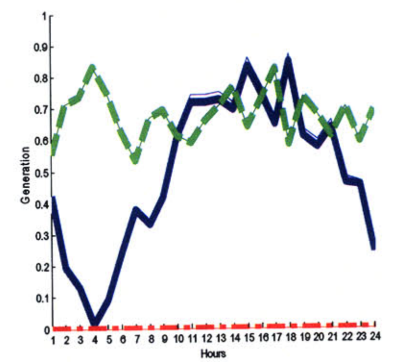
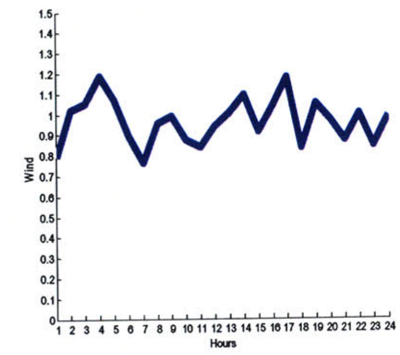
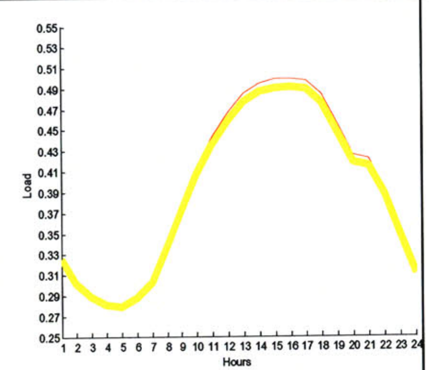
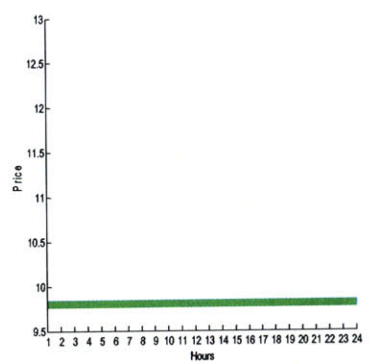
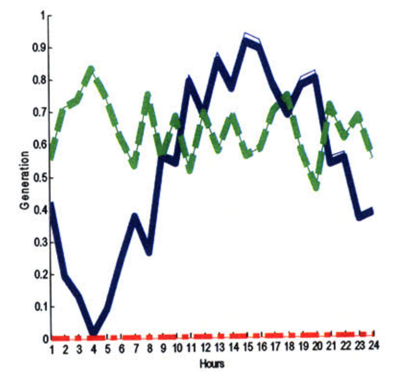
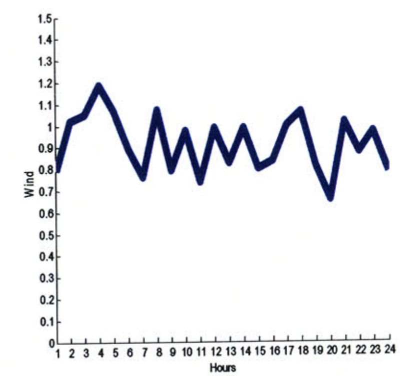
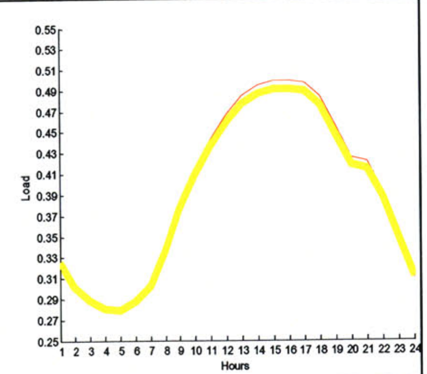
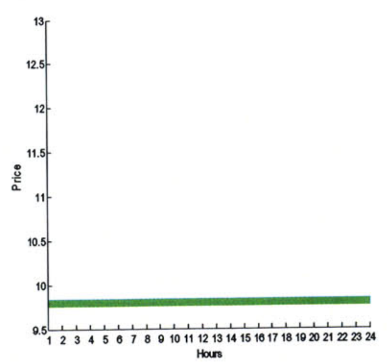
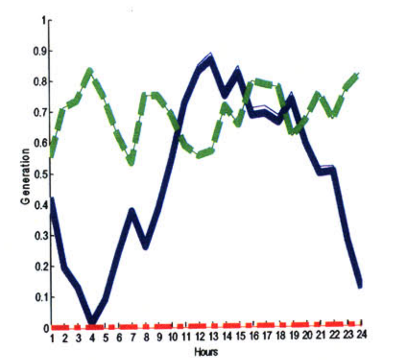
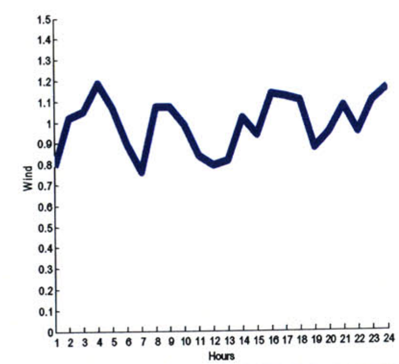
Table 4.5.1 shows the hourly bidding results of the proposed bidding mechanism in the system described above. This table is labeled as "before processed" because oscillation ranges of the non-convergent results are presented instead of the final market equilibrium. In addition, since the hourly biddings only transact the electric use in the future hours, the data of the hours before the transaction hour are left as the reference points of the retailers. Table 4.5.2 shows the hourly bidding results that have the past hours' data updated and all the market equilibrium settled. Section 4.6 will explain in detail how to settle the market equilibriums of the non-convergent cases.

Table 4.5.2 shows that although the market clearing price is lower than the reference prices at most of the hours, the end-users have a lower electric consumption level compared to the reference loads. It is because load shifting to other hours cancels the electric use increment due to a lower price. For the same reason, end users reduce their electric use at peak hours and shift the load to the other hours. Table 4.5.3 shows the hourly bidding results of the SHB. It shows that the end users increase their electric use since the market clearing prices are lower than the reference prices. However, the shifting effects are ignored in the SHB.

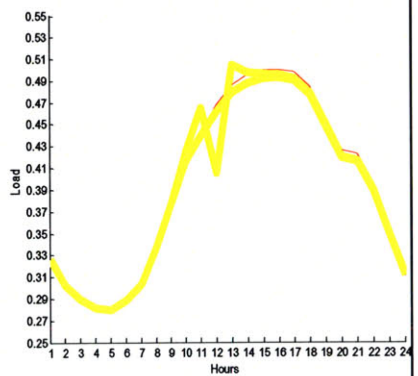
TABLE 4.5.1 The bidding results of the proposed bidding mechanism with renewable energy in the HA market (Before Processed)

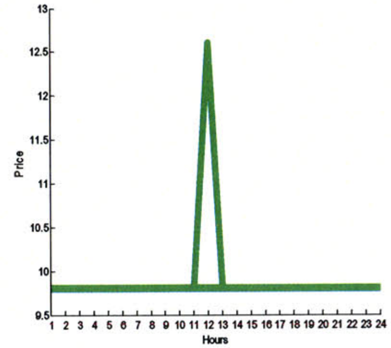
No.	Load profile	Market Clearing Price	Generation Dispatch Schedule	Wind Capacity
H1				
H2				
H3				

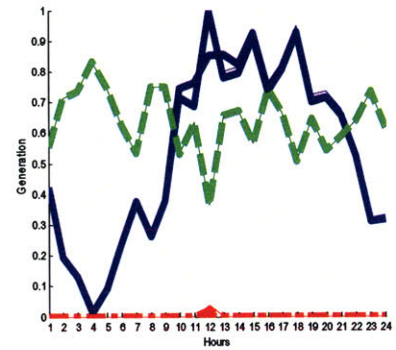

H4
H5
H6
Load
Price
Generation
Wind
Hours

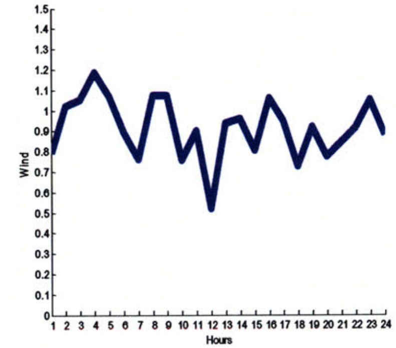
H7				
H8				
H9				

H10

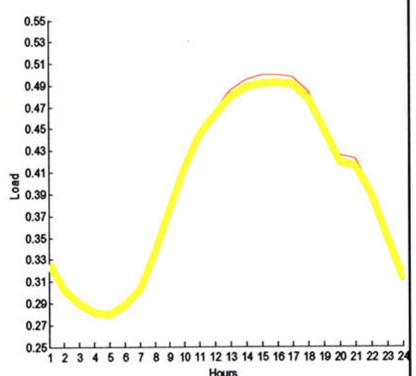

Load
Hours

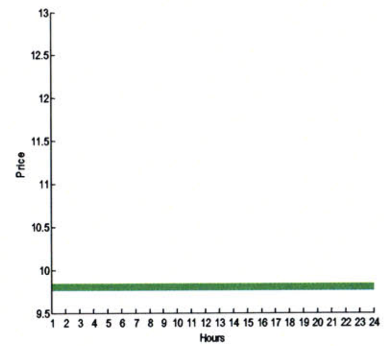

Price
Hours

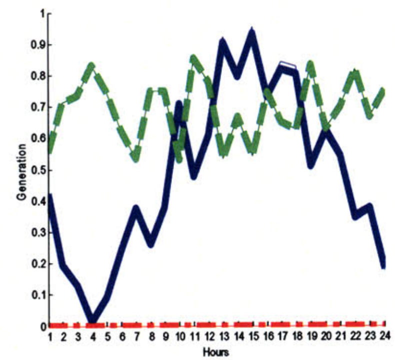

Generation
Hours

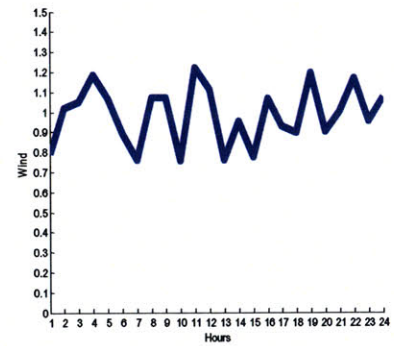

Wind
Hours

H11

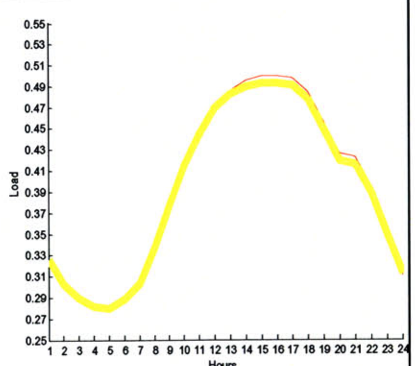

Load
Hours

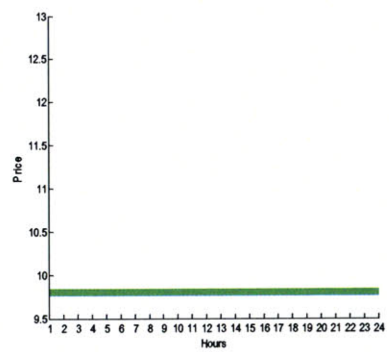

Price
Hours

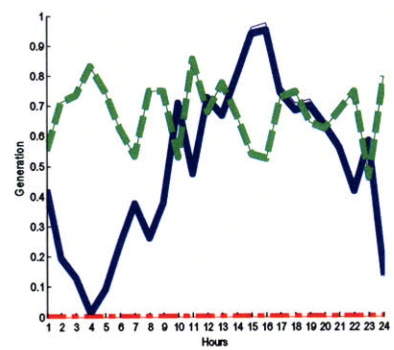

Generation
Hours

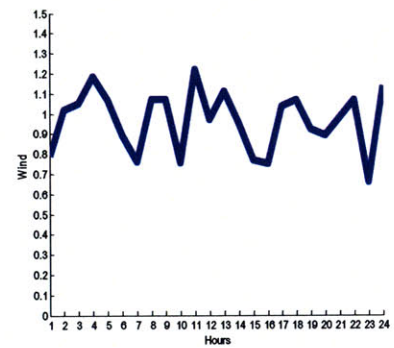

Wind
Hours

H12

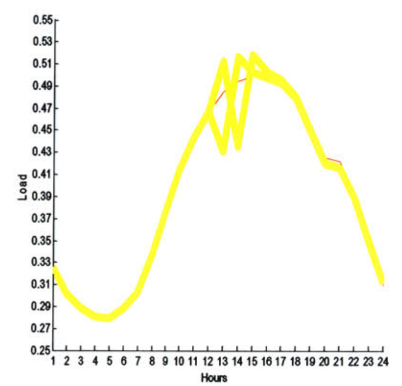

Load
Hours

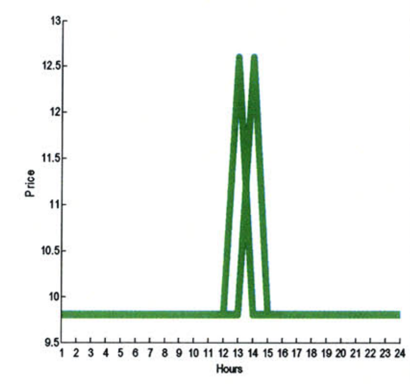

Price
Hours

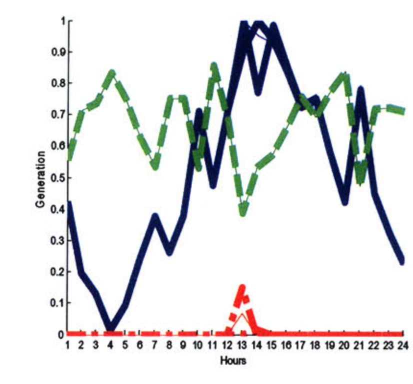

Generation
Hours

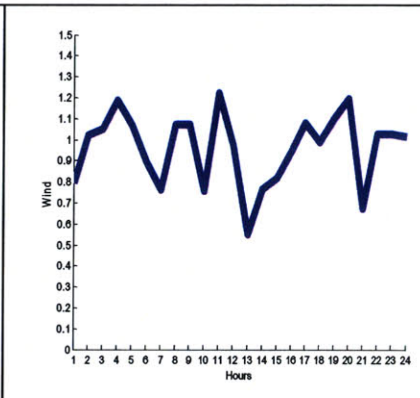

Wind
Hours

H13

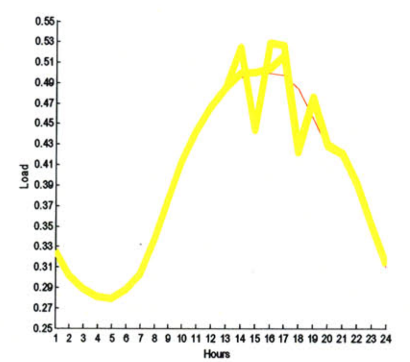

Load
Hours

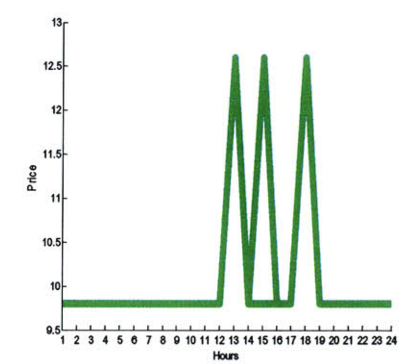

Price
Hours

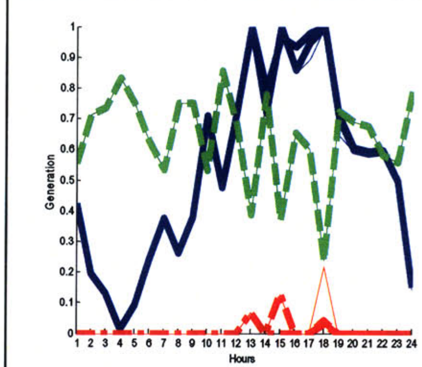

Generation
Hours

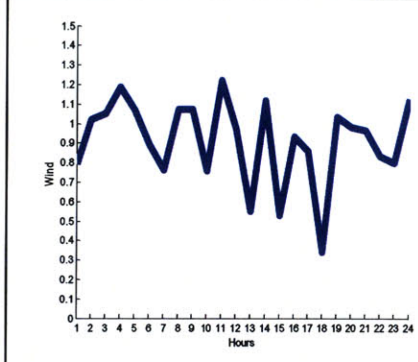

Wind
Hours

H14

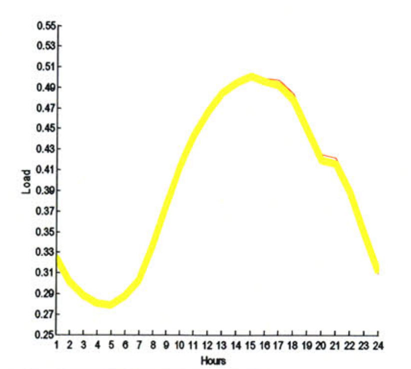

Load
Hours

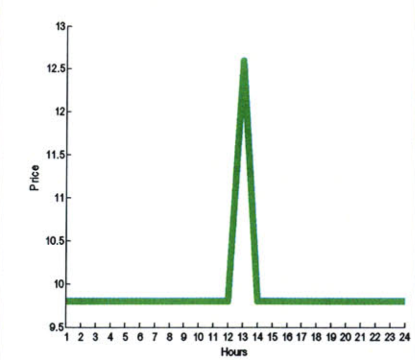

Price
Hours

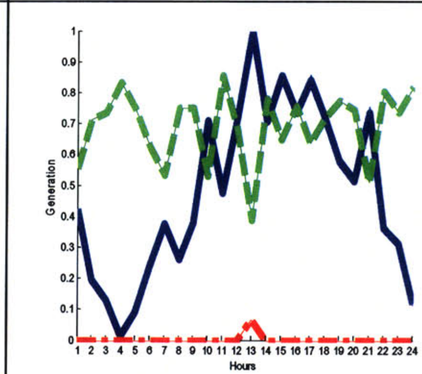

Generation
Hours

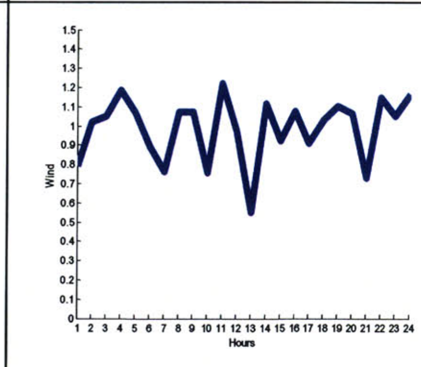

Wind
Hours

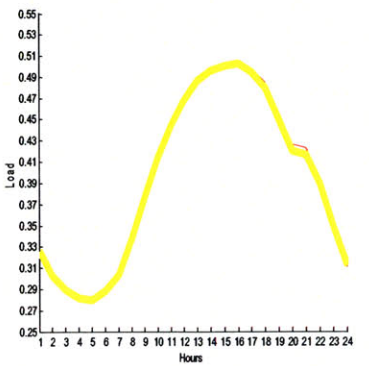
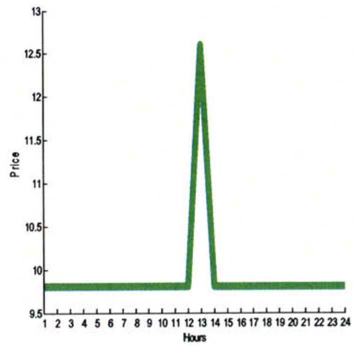
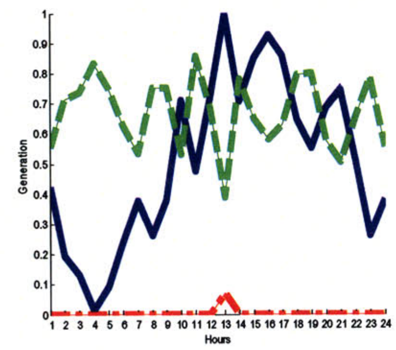
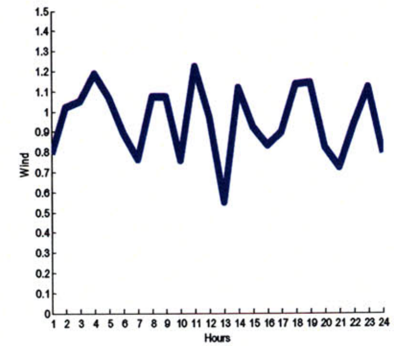
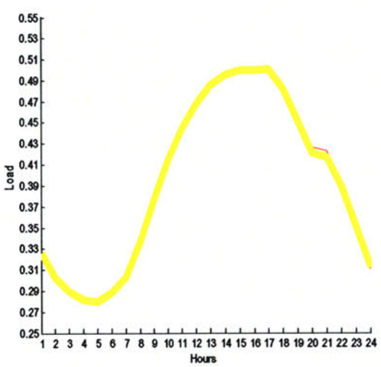
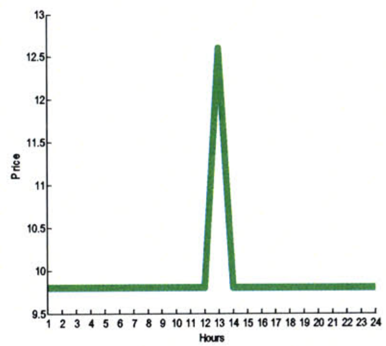
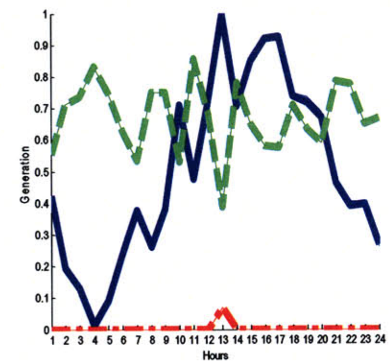
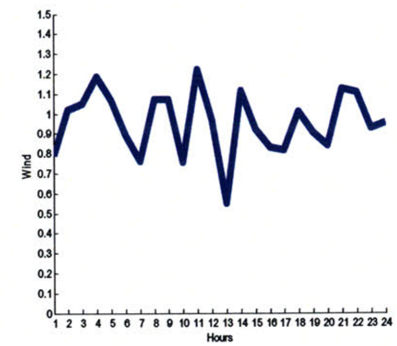
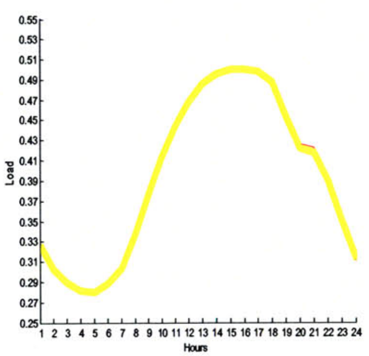
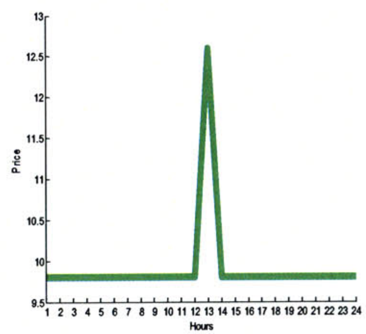
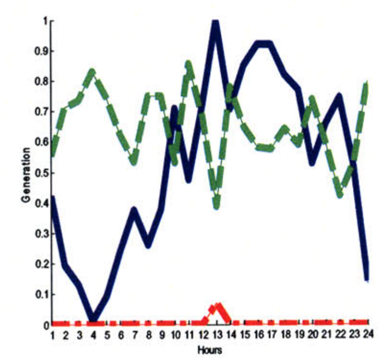
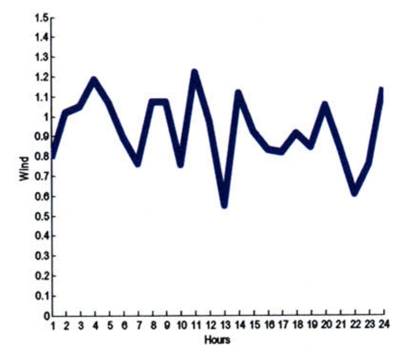
H15

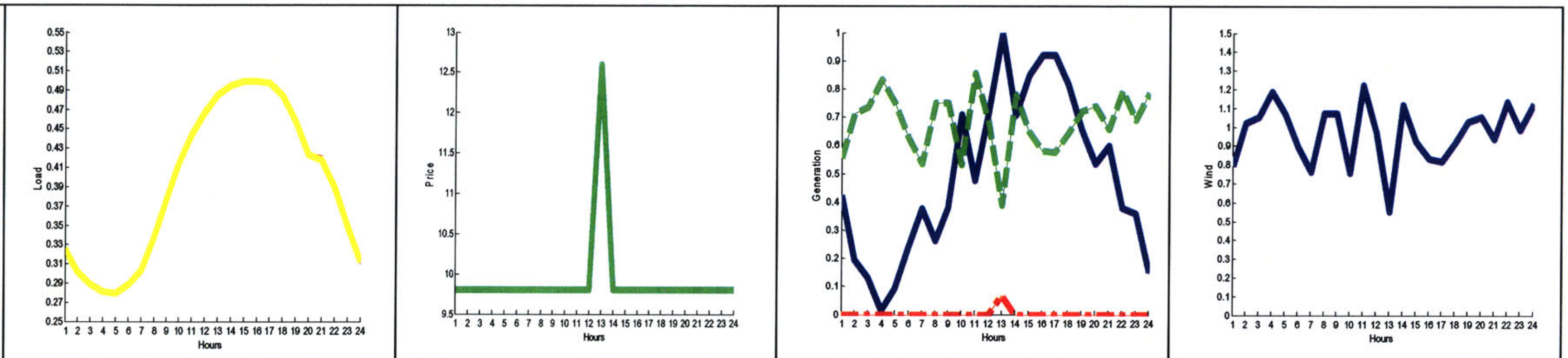
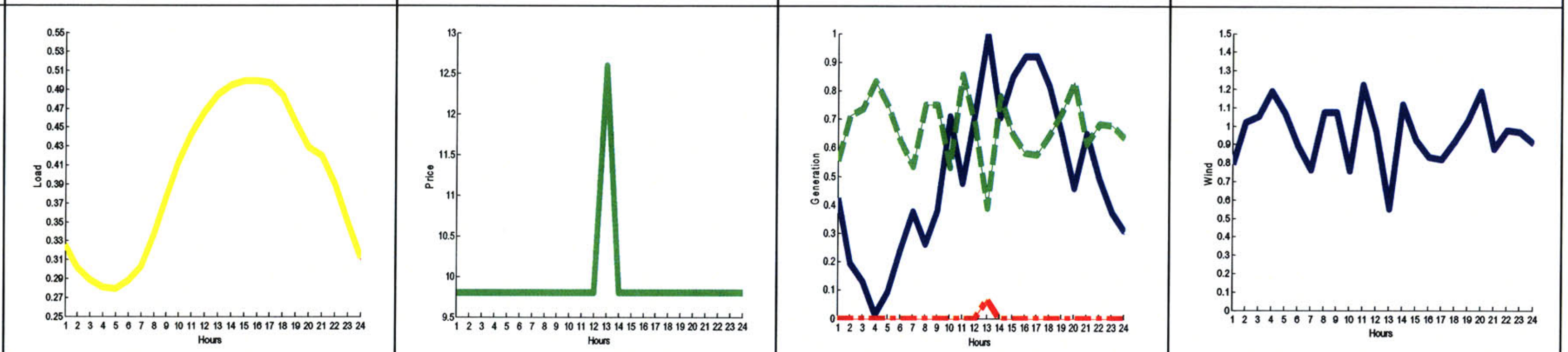
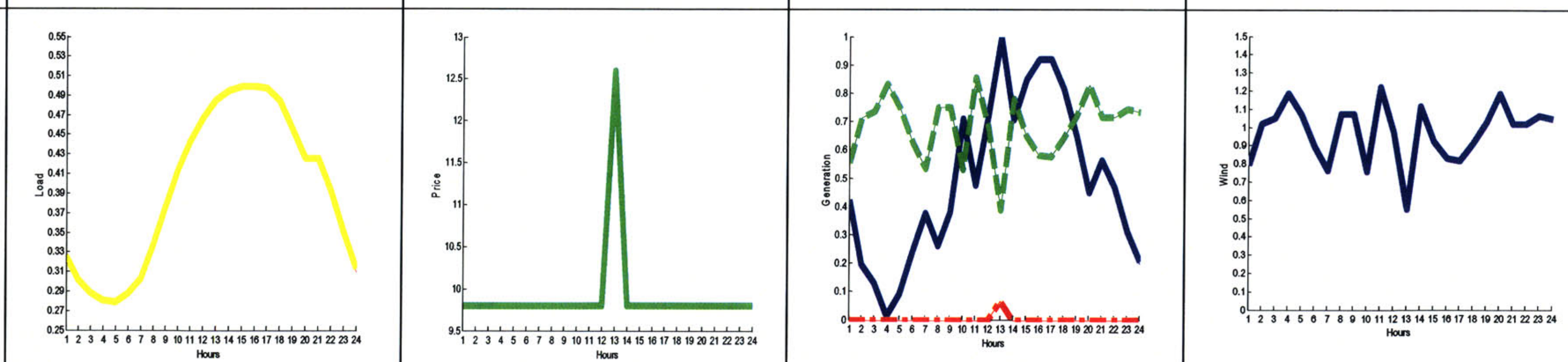

Load
Hours


Price
Hours


Generation
Hours


Wind
Hours

H16	Load Hours	Price Hours	Generation Hours	Wind Hours
H17	Load Hours	Price Hours	Generation Hours	Wind Hours
H18	Load Hours	Price Hours	Generation Hours	Wind Hours

H19	Load Hours Price Hours Generation Hours Wind Hours
H20	Load Hours Price Hours Generation Hours Wind Hours
H21	Load Hours Price Hours Generation Hours Wind Hours

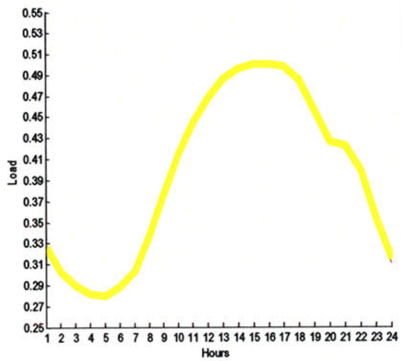
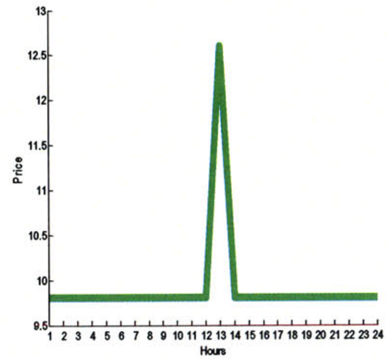
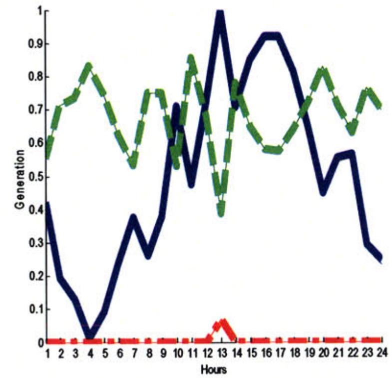
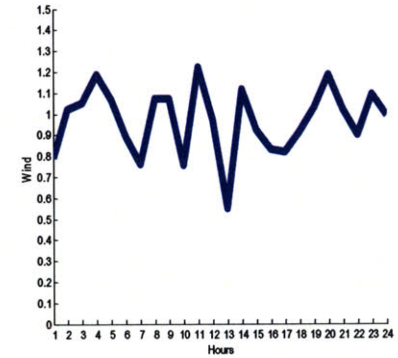
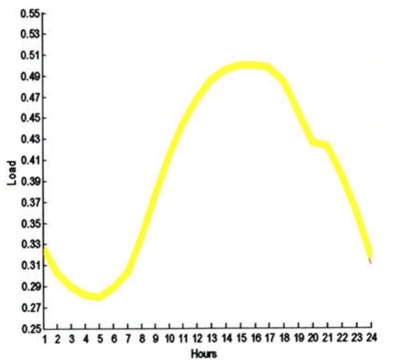
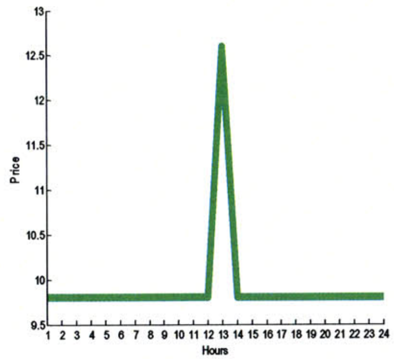
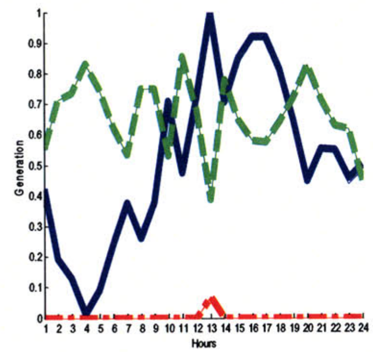
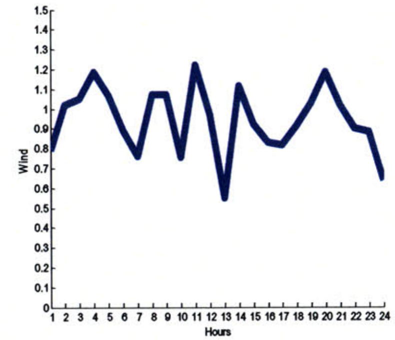
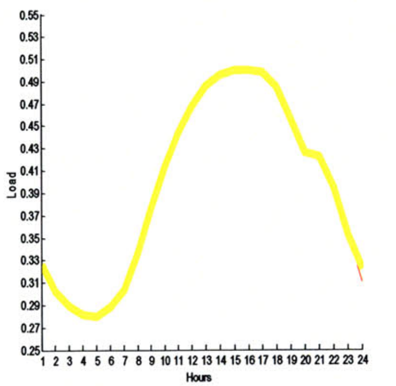
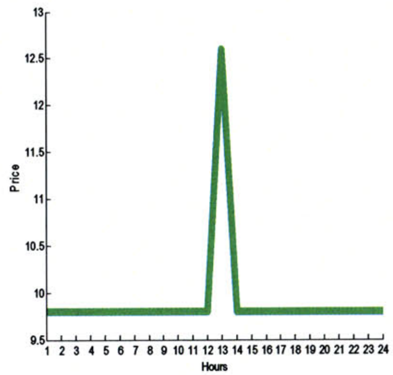
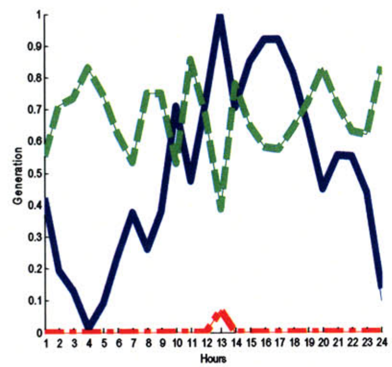
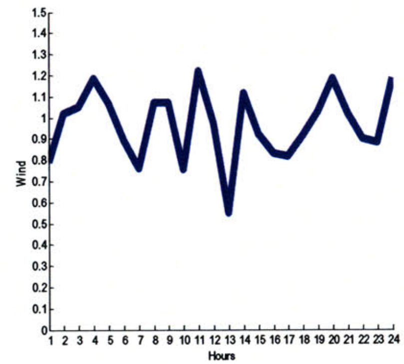
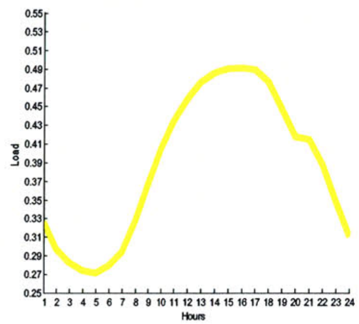
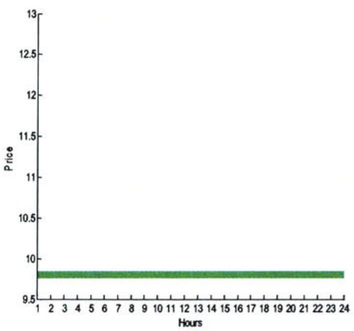
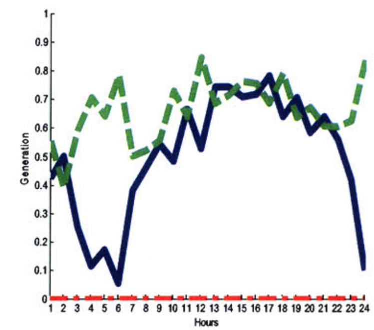
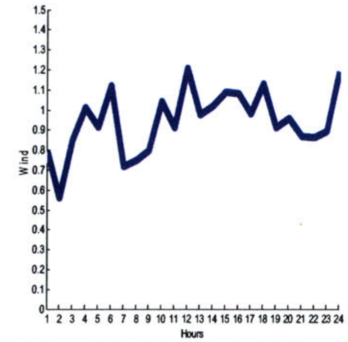
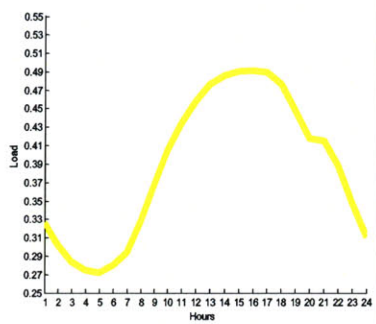
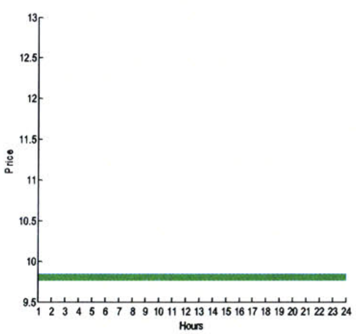
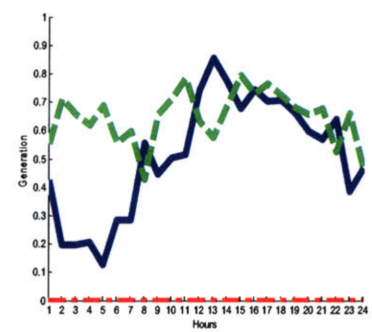
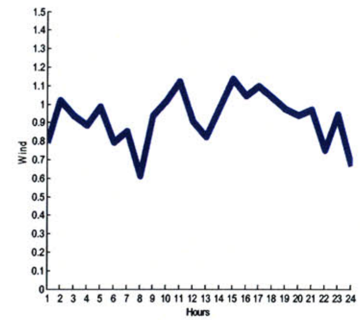
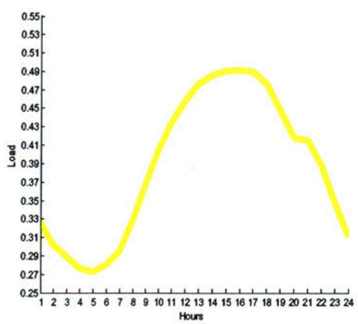
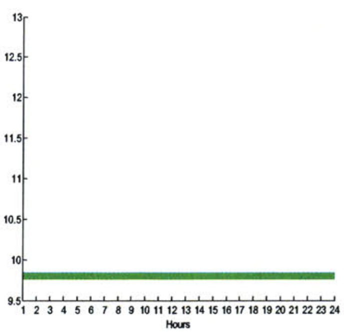
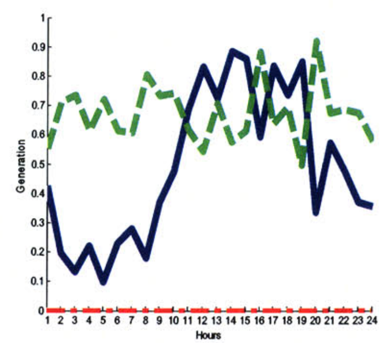
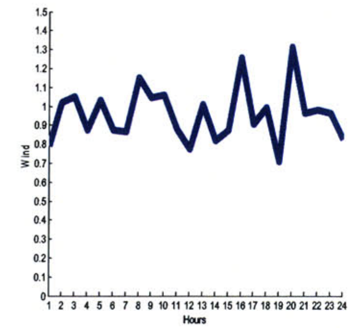
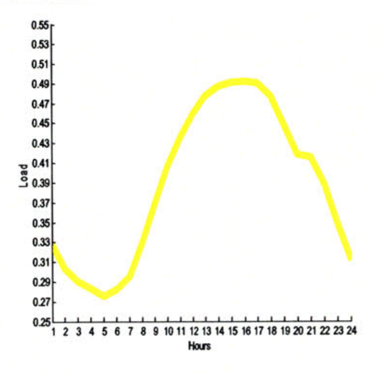
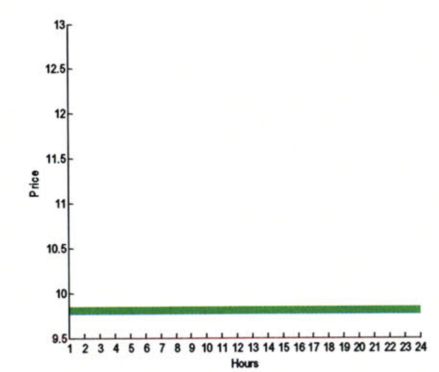
H22	Load Hours	Price Hours	Generation Hours	Wind Hours
H23	Load Hours	Price Hours	Generation Hours	Wind Hours
H24	Load Hours	Price Hours	Generation Hours	Wind Hours

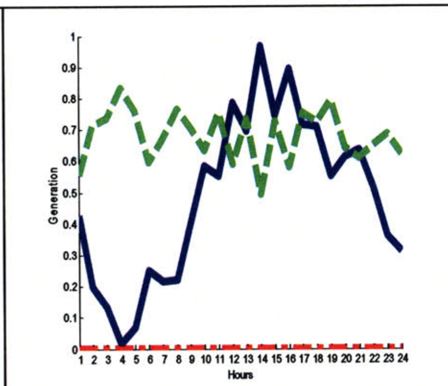
TABLE 4.5.2 The bidding results of the proposed bidding mechanism with renewable energy in the HA market (After Processed)

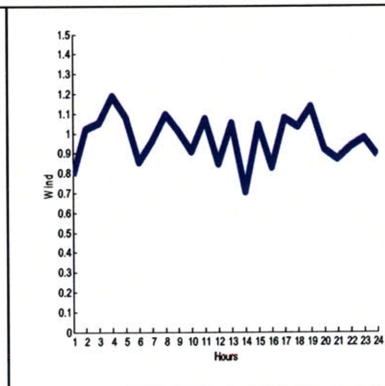
No.	Load profile	Market Clearing Price	Generation Dispatch Schedule	Wind Capacity
H1				
H2				
H3				

H4

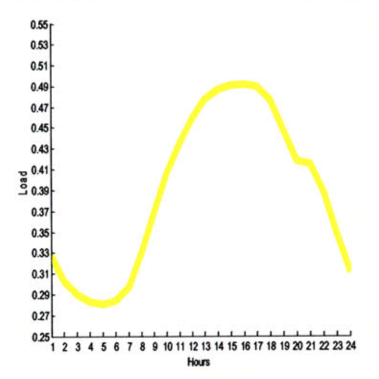

Load
Hours

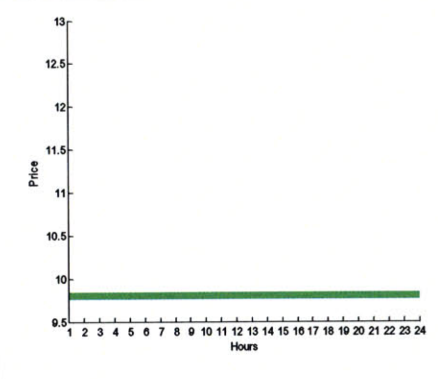

Price
Hours

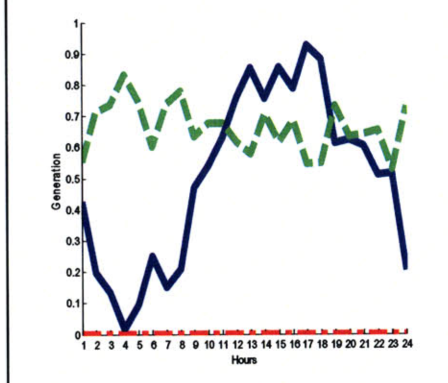

Generation
Hours

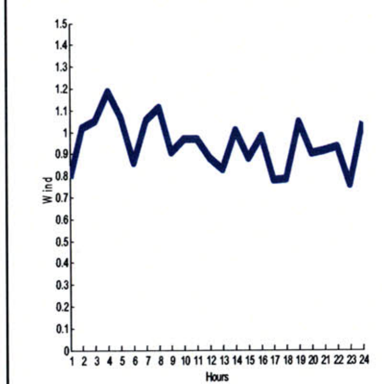

Wind
Hours

H5

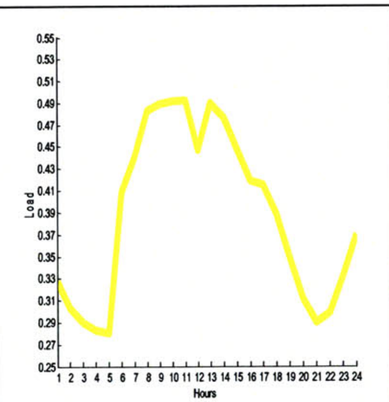

Load
Hours

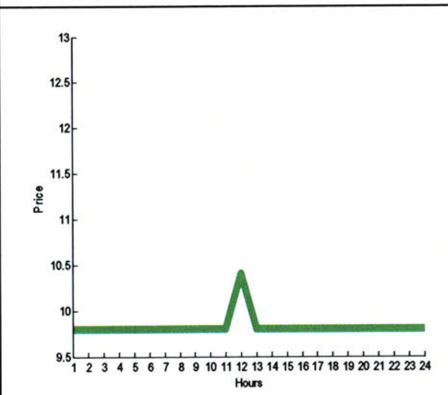

Price
Hours

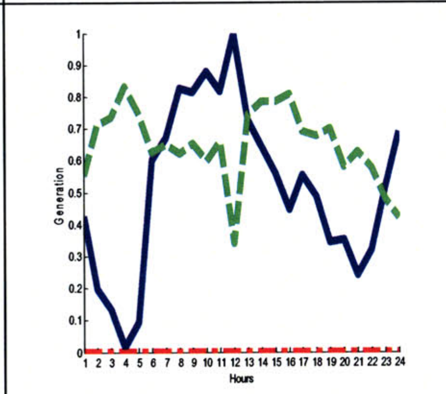

Generation
Hours

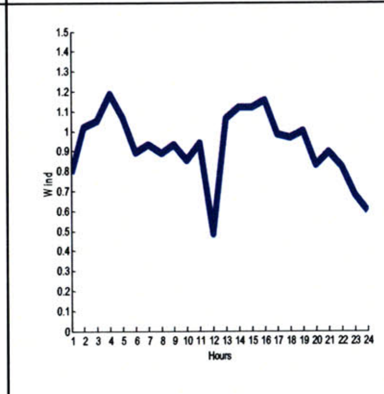

Wind
Hours

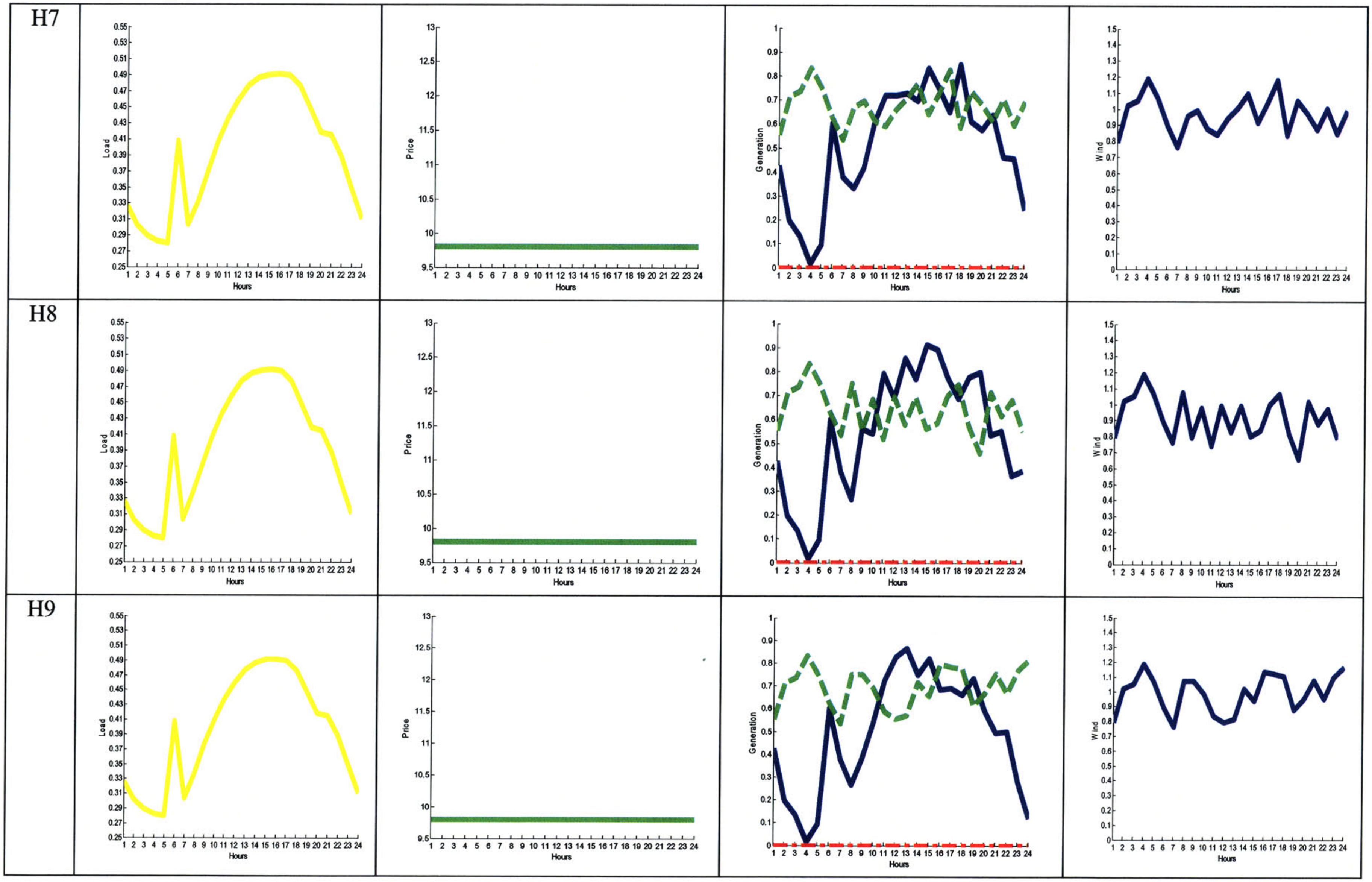
H6


Load
Hours

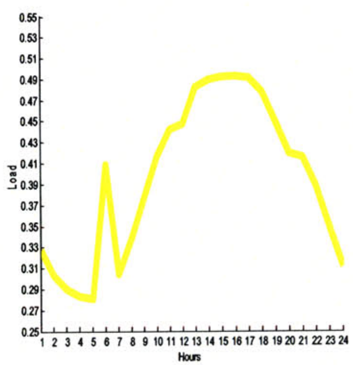

Price
Hours

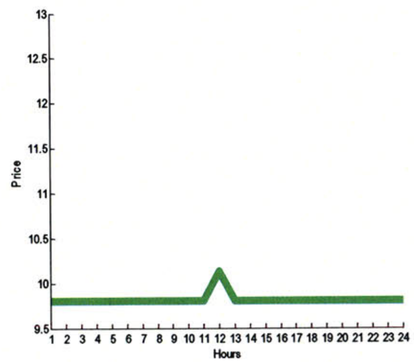

Generation
Hours

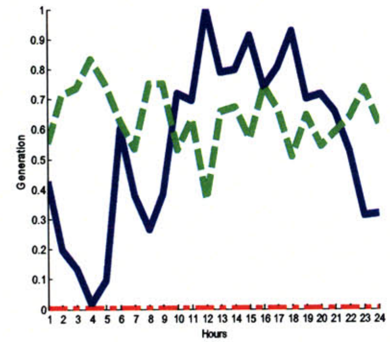

Wind
Hours

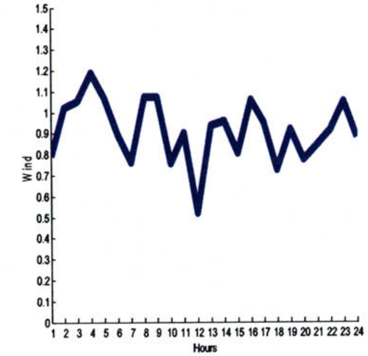

H7
H8
H9
Load
Price
Generation
Wind
Hours

H10

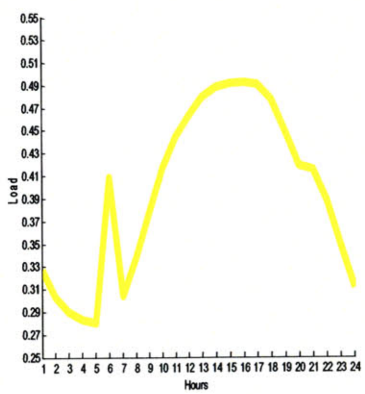

Load
Hours

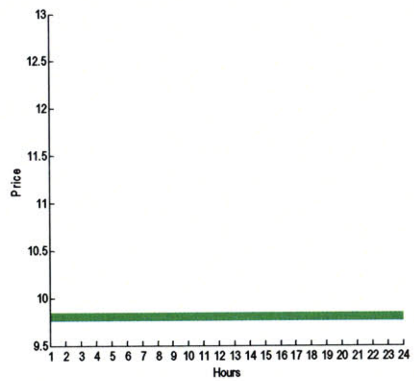

Price
Hours

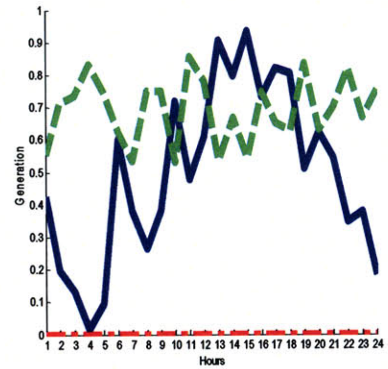

Generation
Hours

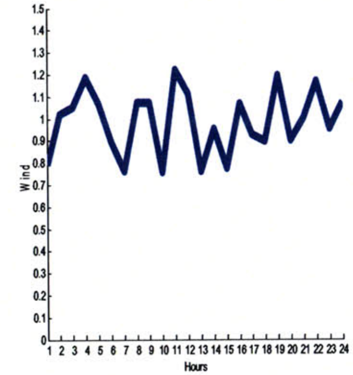

Wind
Hours

H11

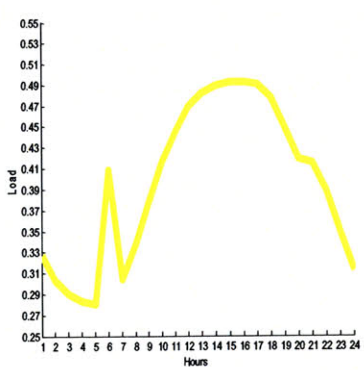

Load
Hours

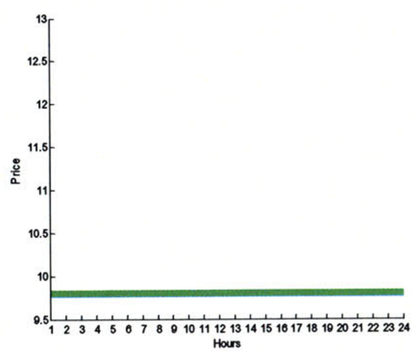

Price
Hours

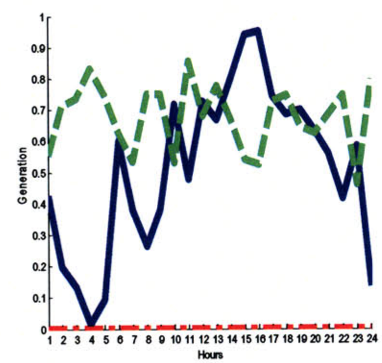

Generation
Hours

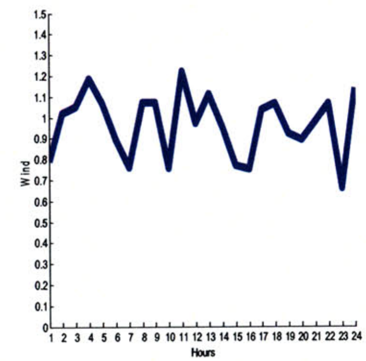

Wind
Hours

H12

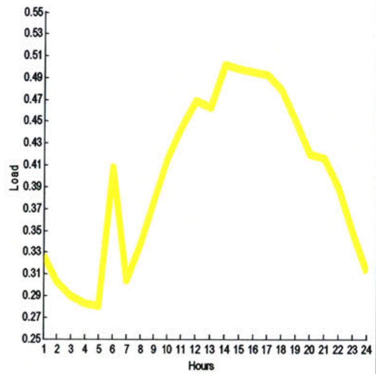

Load
Hours

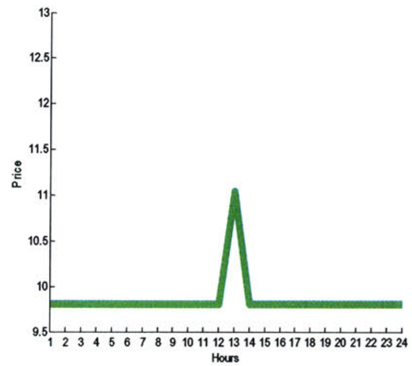

Price
Hours

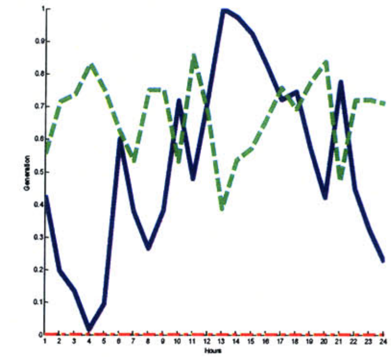

Generation
Hours

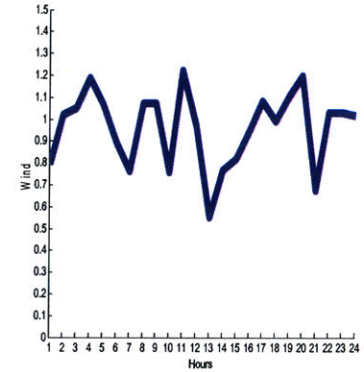

Wind
Hours

H13

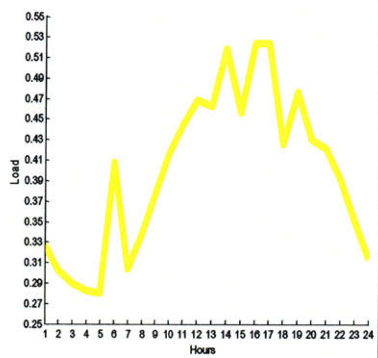

Load
Hours

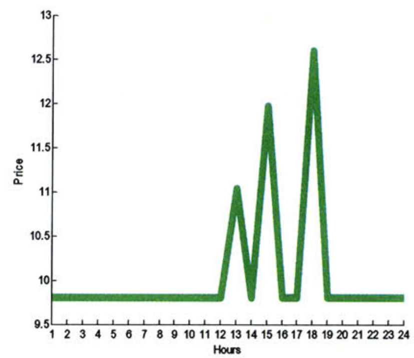

Price
Hours

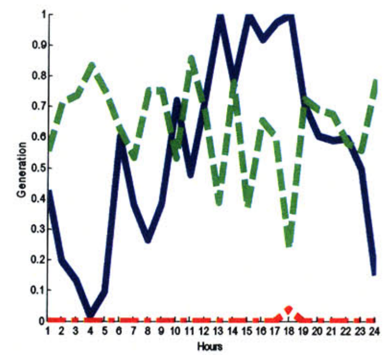


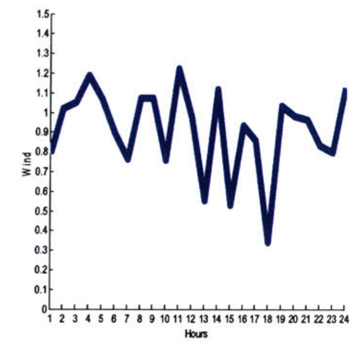

Wind
Hours

H14

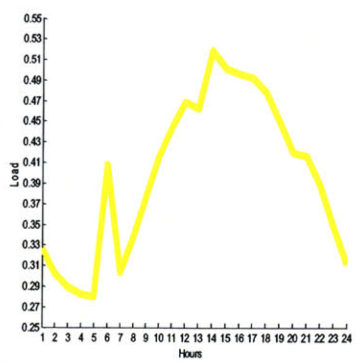

Load
Hours

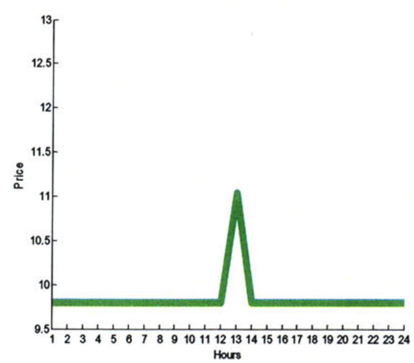

Price
Hours

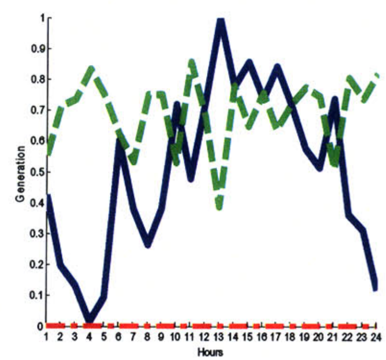

Generation
Hours

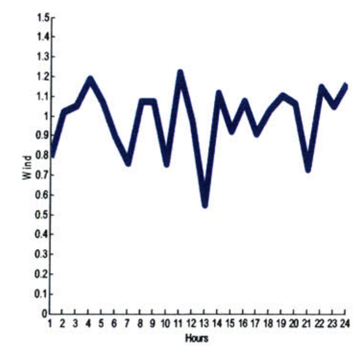

Wind
Hours

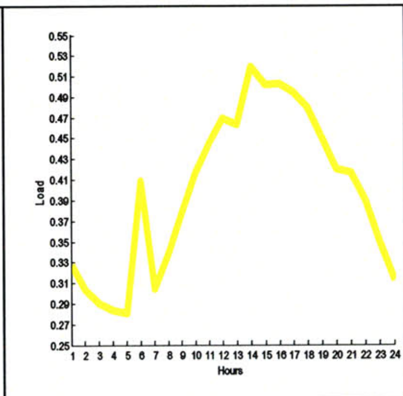
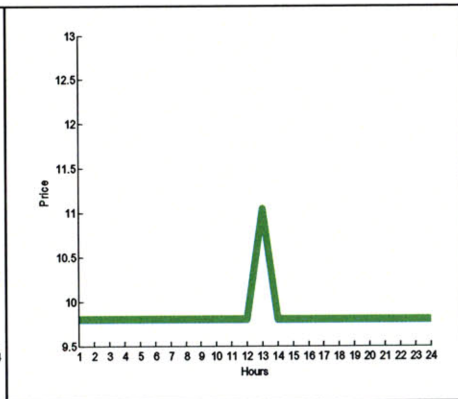
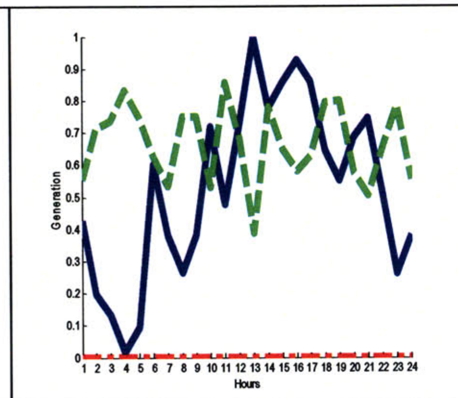
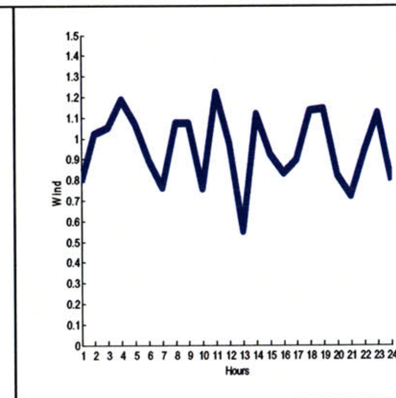
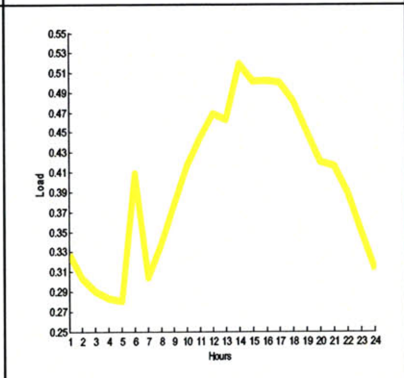
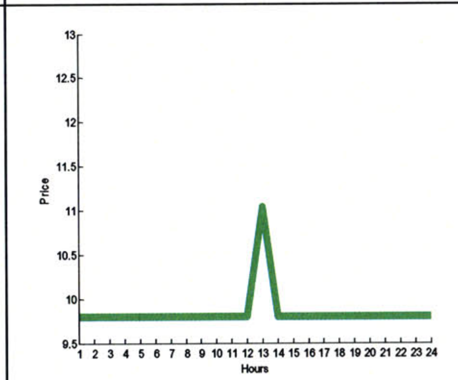
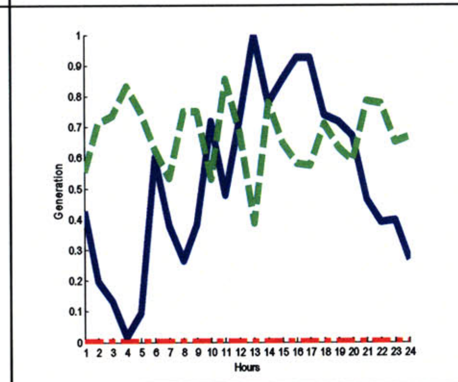
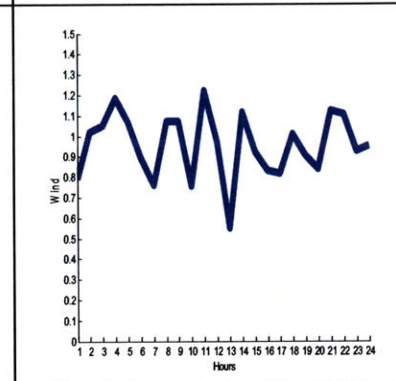
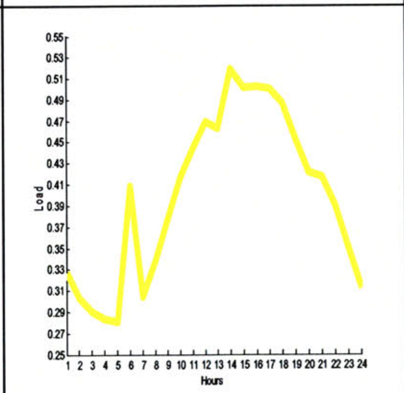
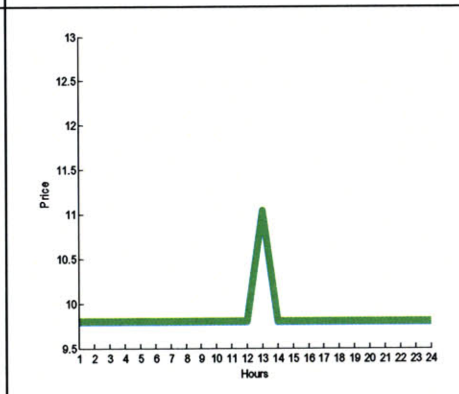
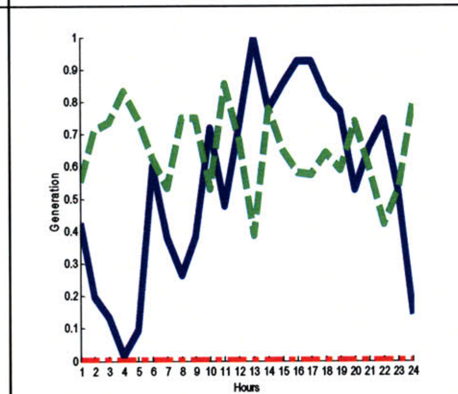
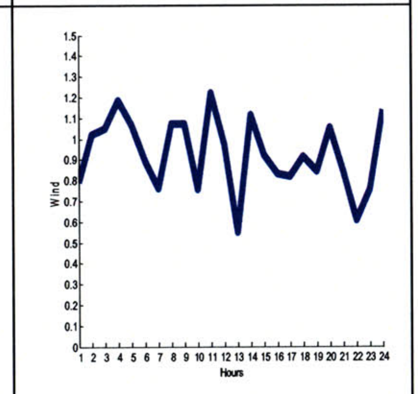
H15


Load
Hours

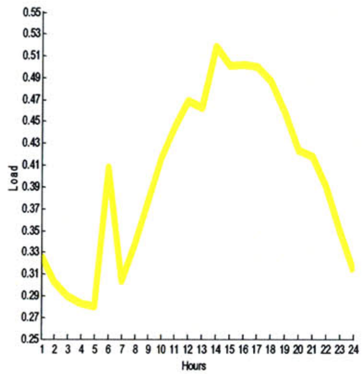

Price
Hours

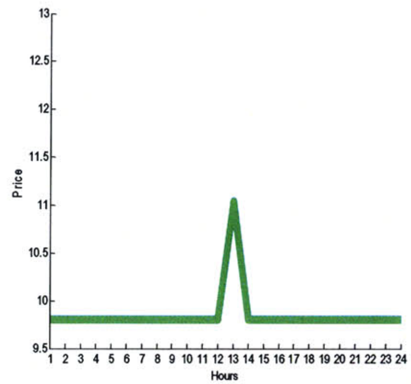

Generation
Hours

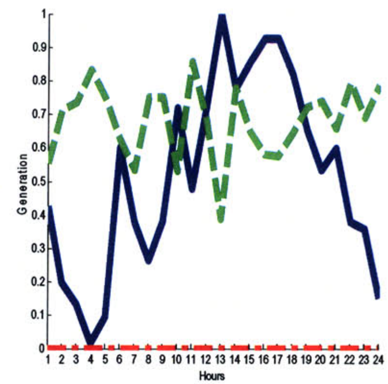

Wind
Hours

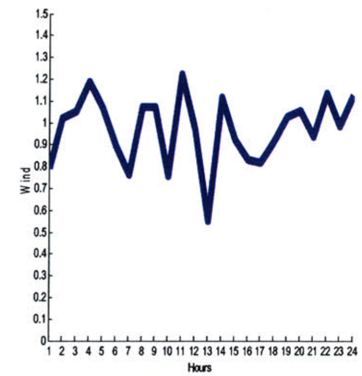
H16	Load Hours	Price Hours	Generation Hours	Wind Hours
H17	Load Hours	Price Hours	Generation Hours	Wind Hours
H18	Load Hours	Price Hours	Generation Hours	Wind Hours

H19

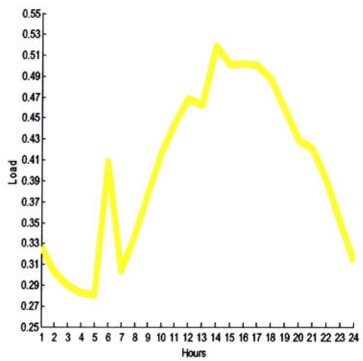

Load
Hours

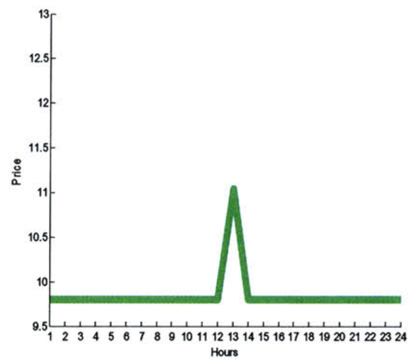

Price
Hours

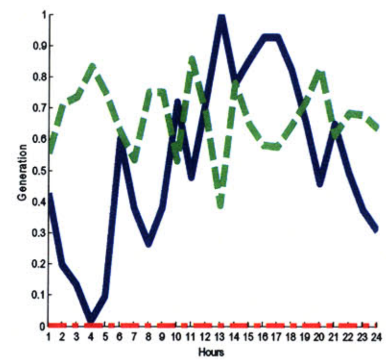

Generation
Hours

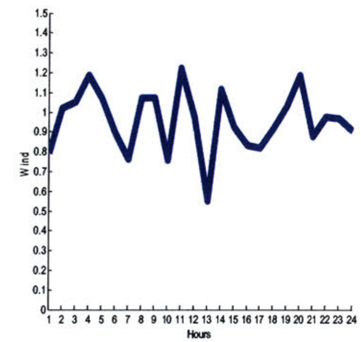

Wind
Hours

H20

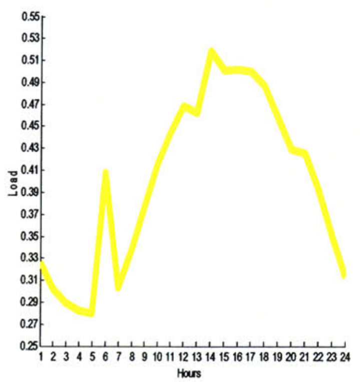

Load
Hours

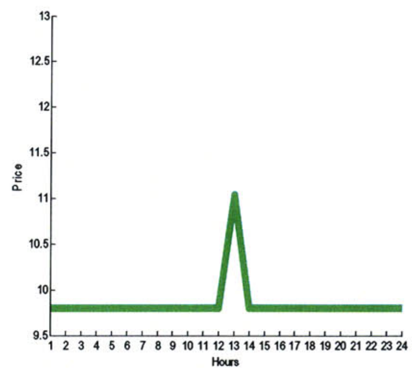

Price
Hours

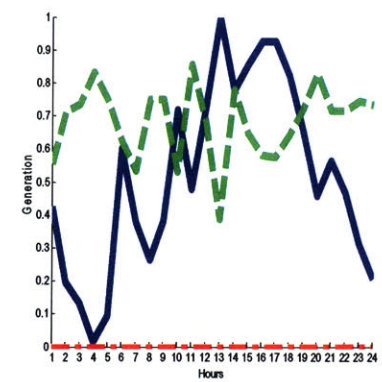

Generation
Hours

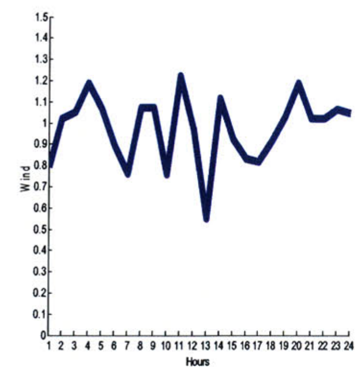

Wind
Hours

H21

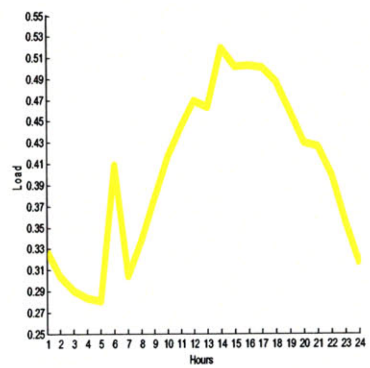

Load
Hours

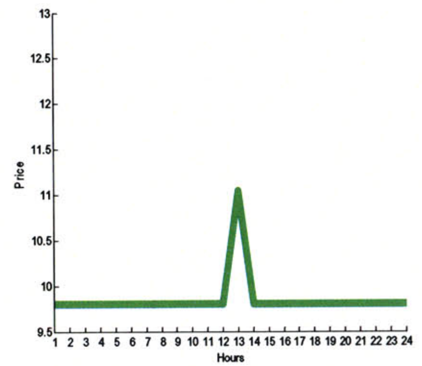

Price
Hours

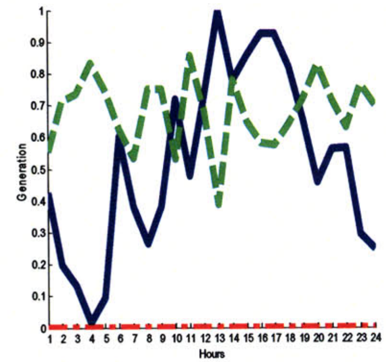

Generation
Hours

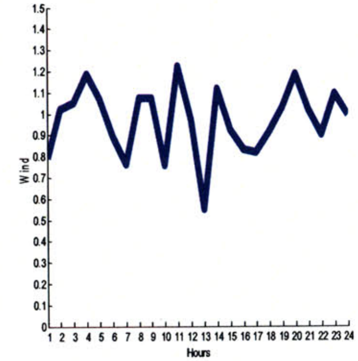

Wind
Hours

H22

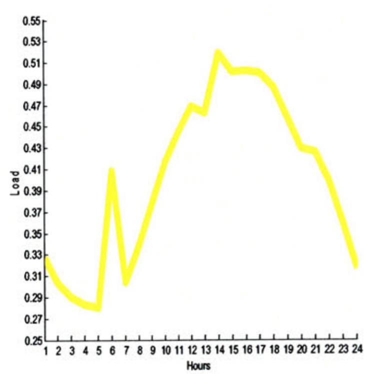

Load
Hours

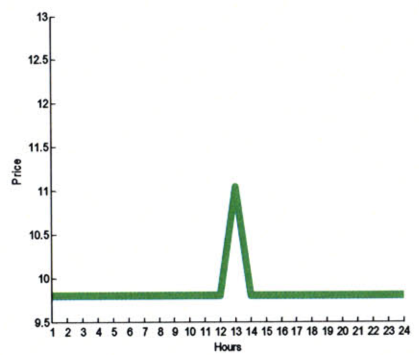

Price
Hours

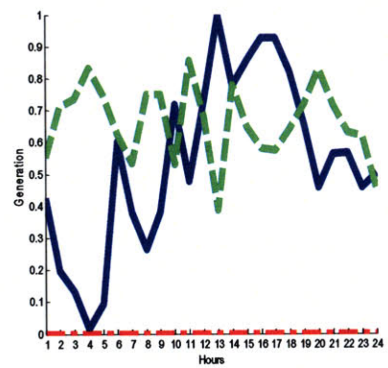

Generation
Hours

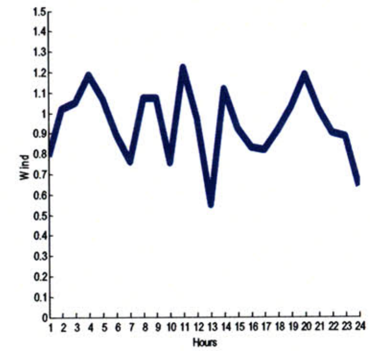

Wind
Hours

H23

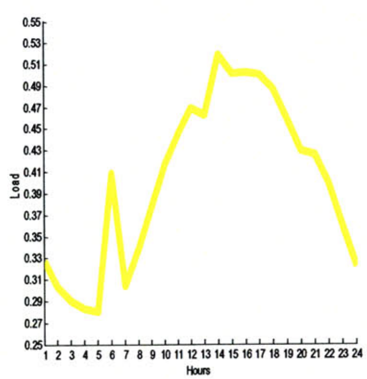

Load
Hours

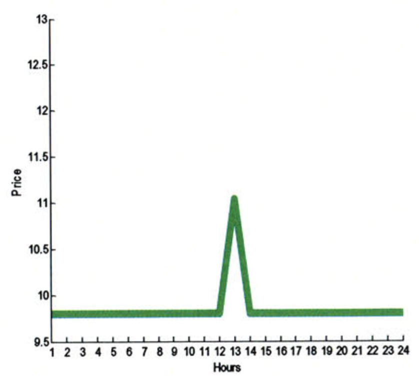

Price
Hours

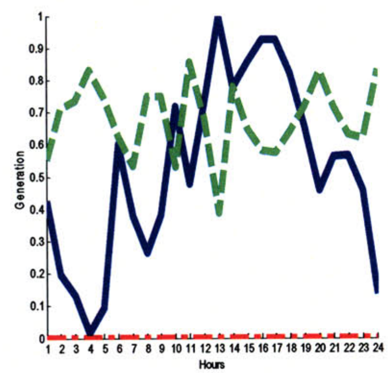

Generation
Hours


Wind
Hours

H24


Load
Hours


Price
Hours


Generation
Hours

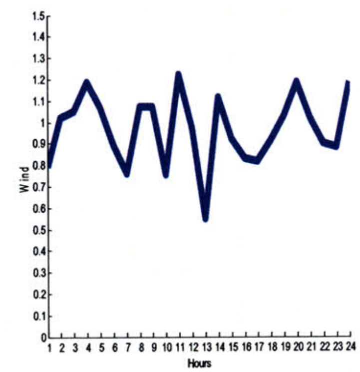
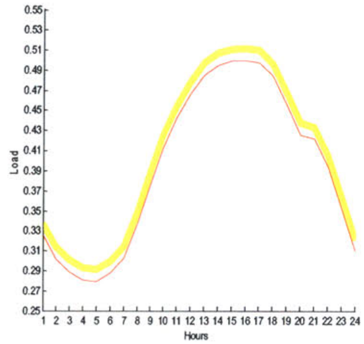
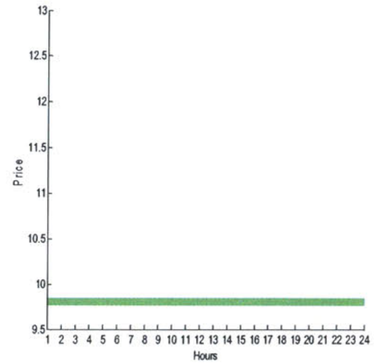
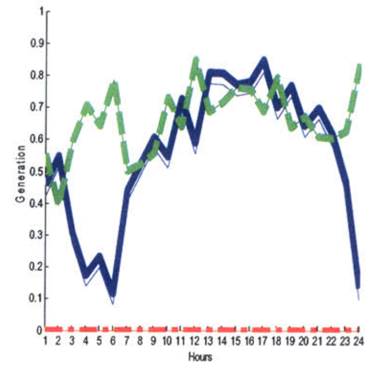
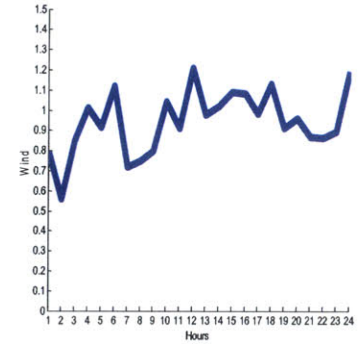
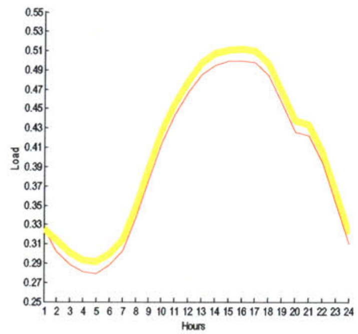
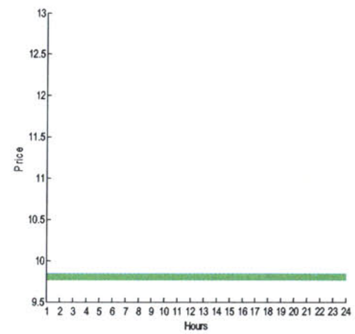
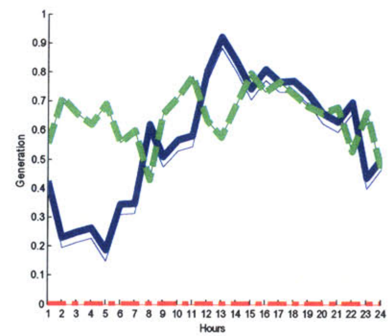
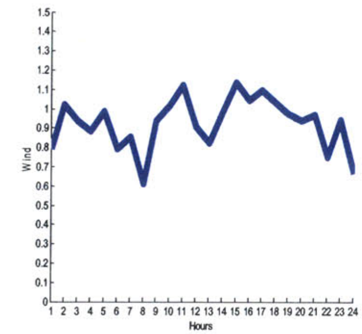
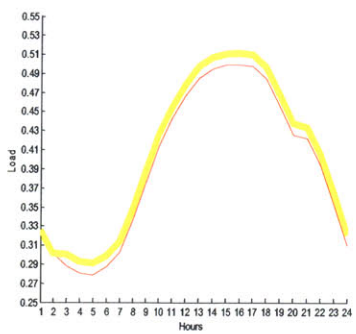
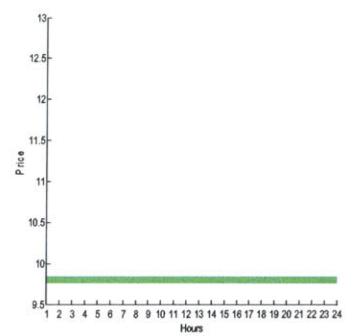
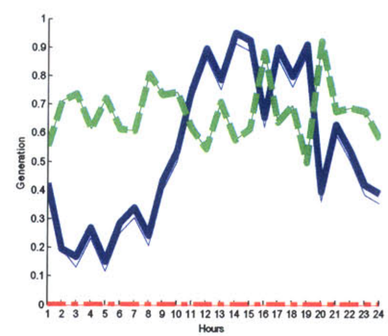
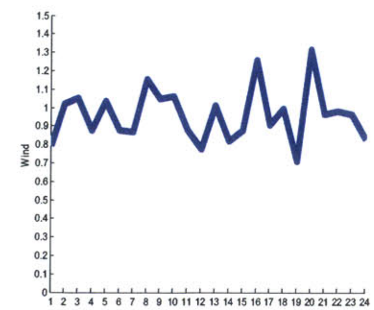
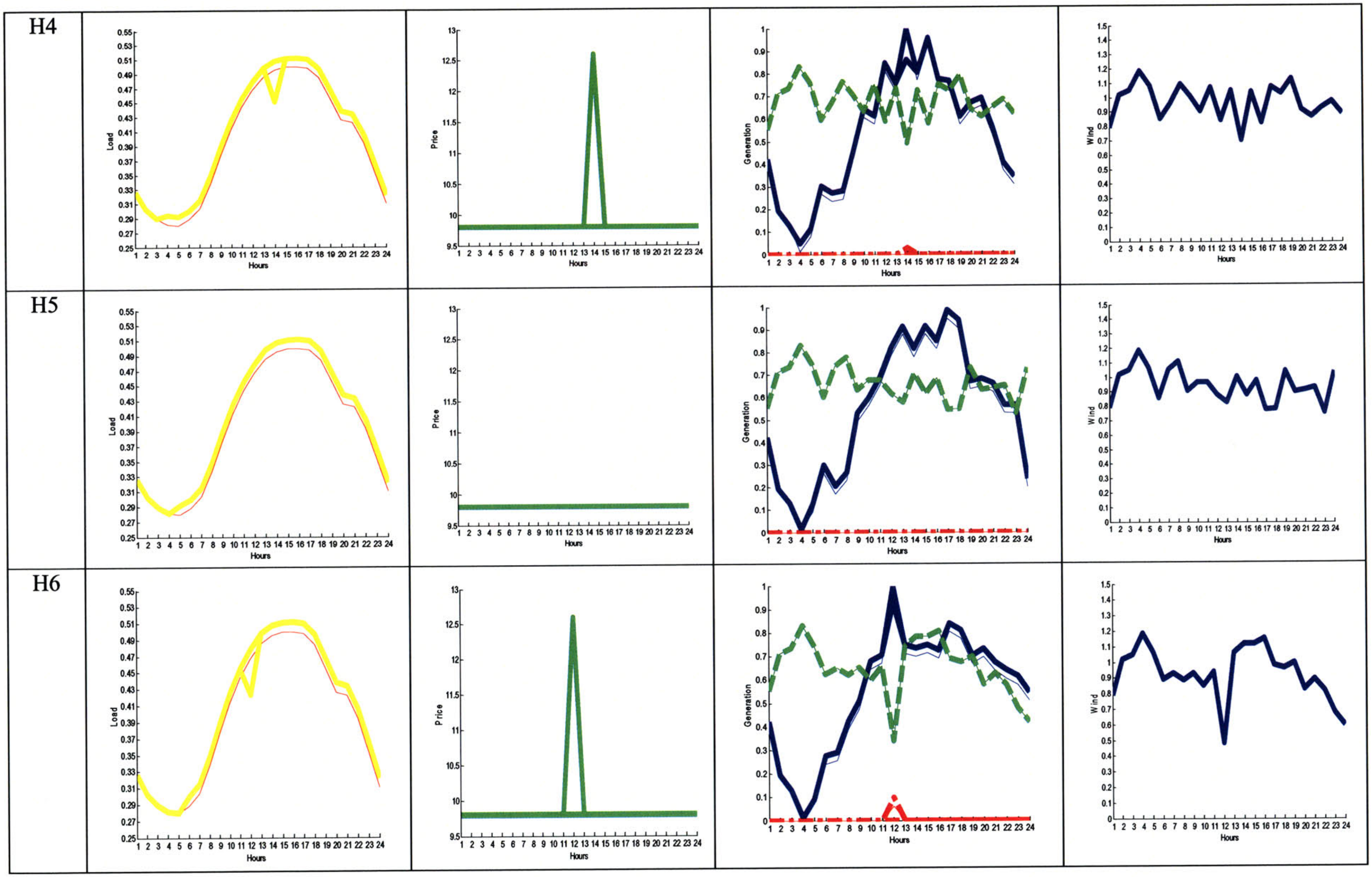
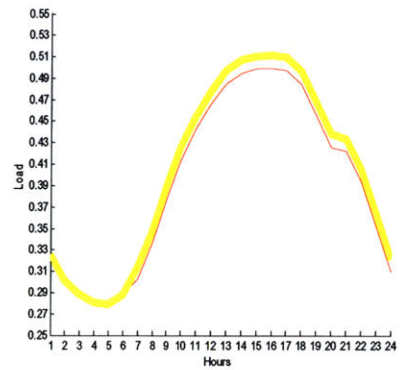
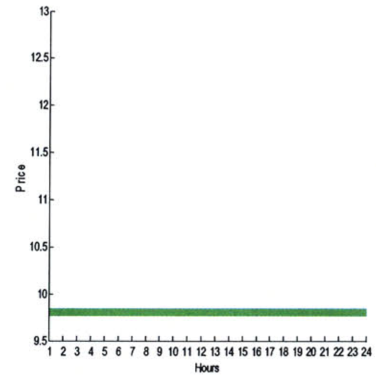
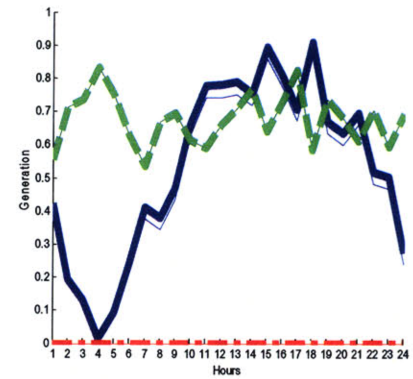
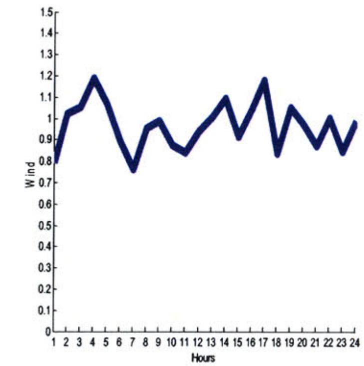
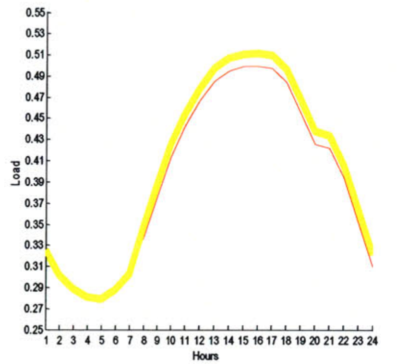
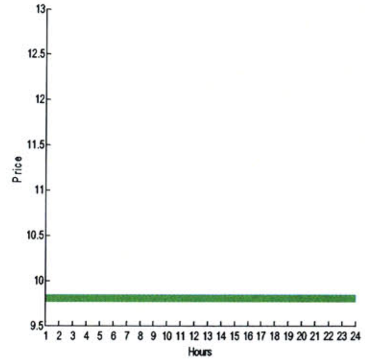
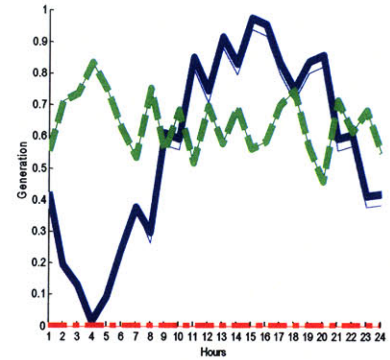
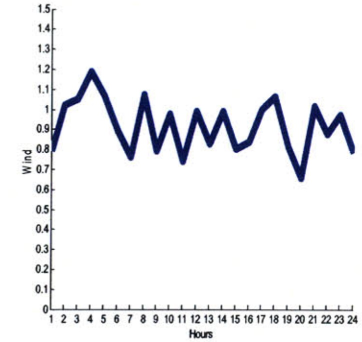
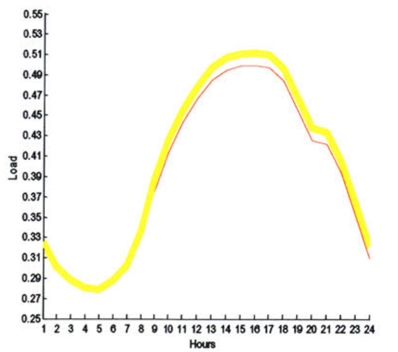
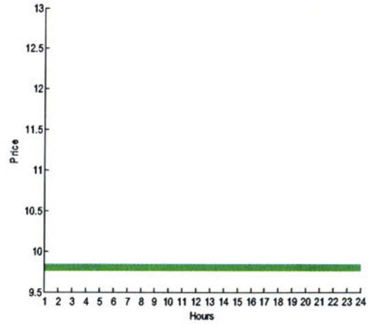
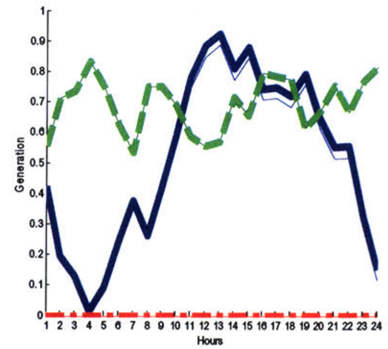
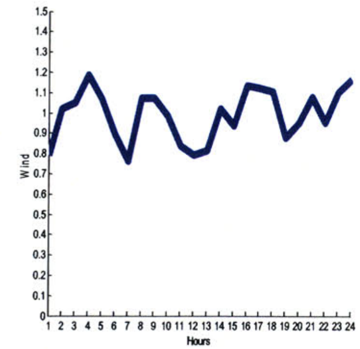
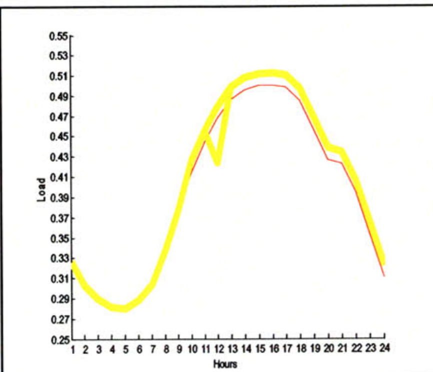
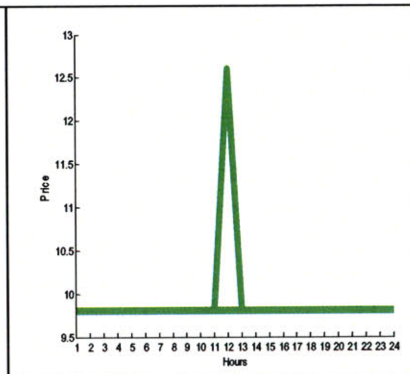
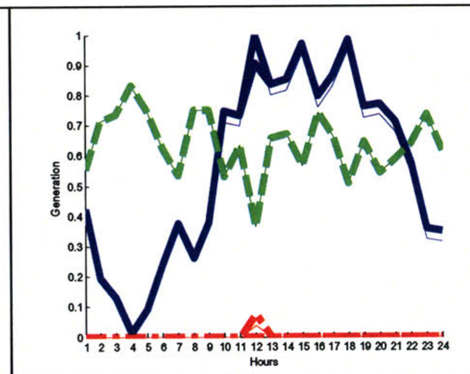
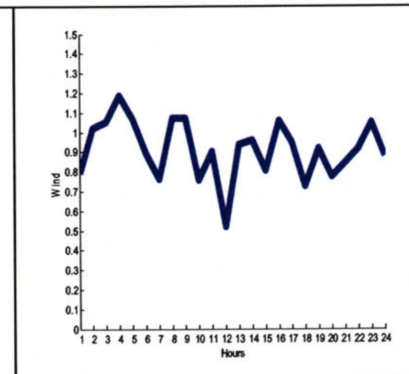
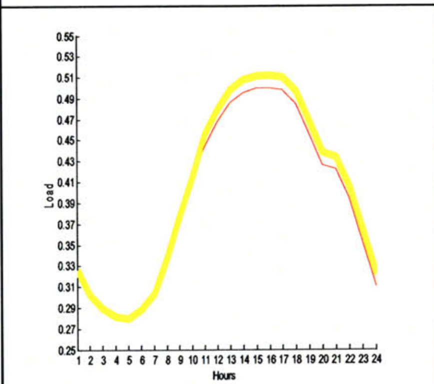
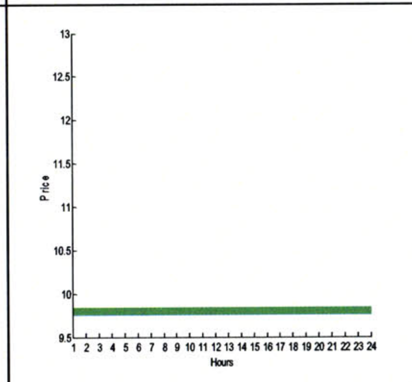
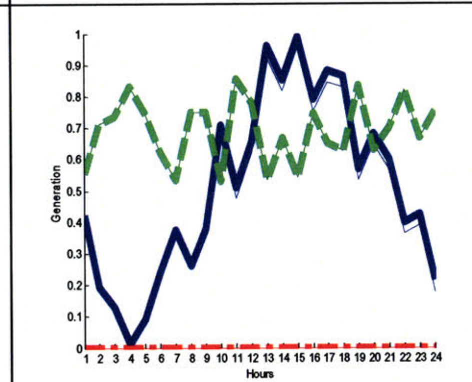
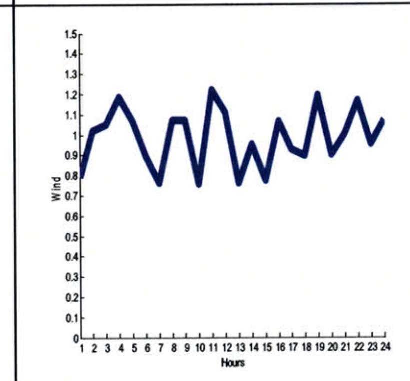
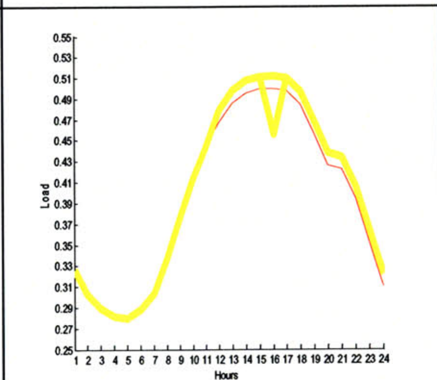
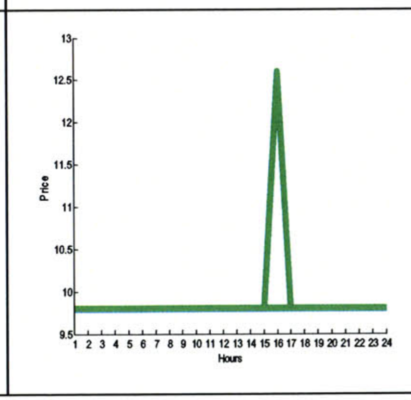
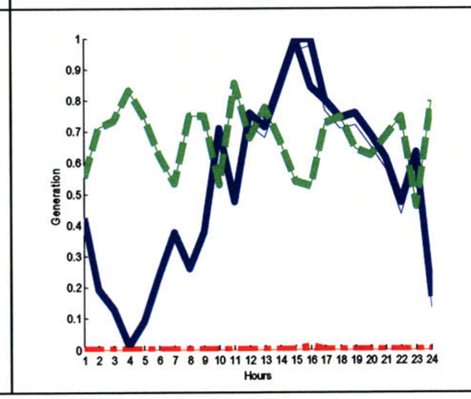
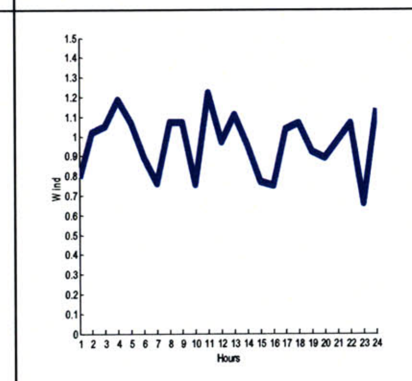

Wind
Hours

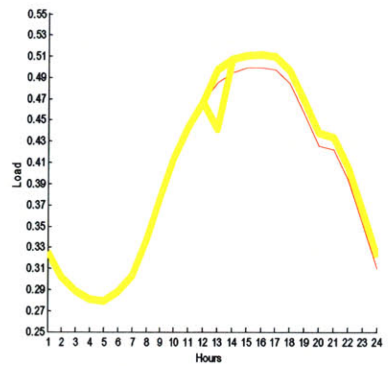
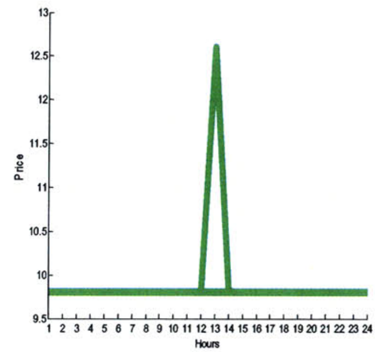
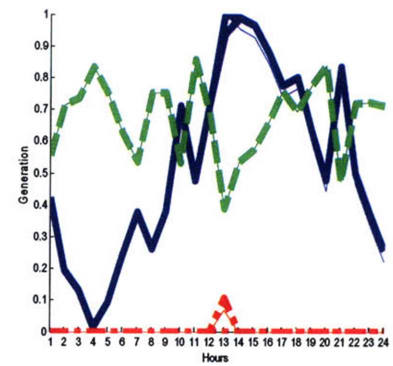
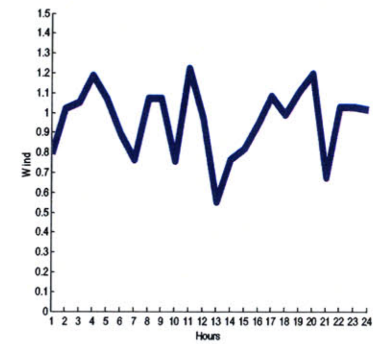
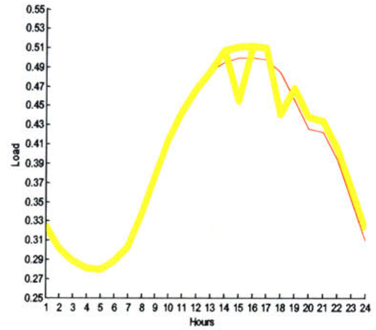
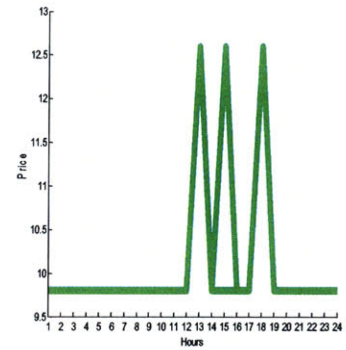
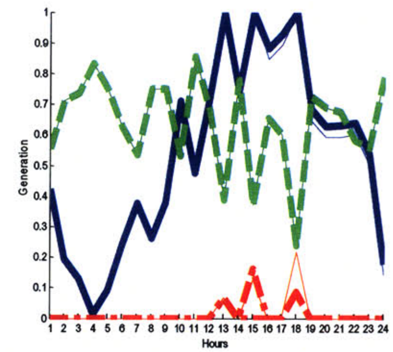
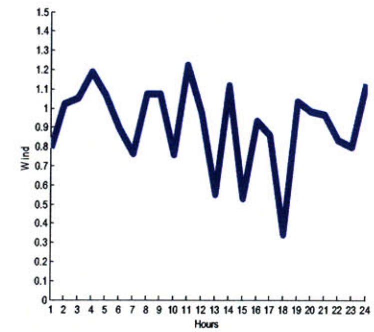
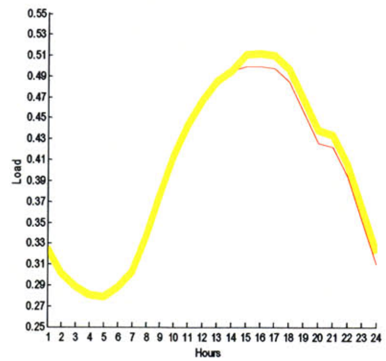
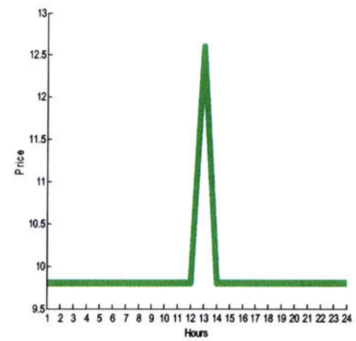
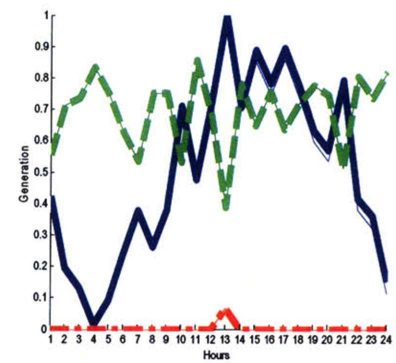
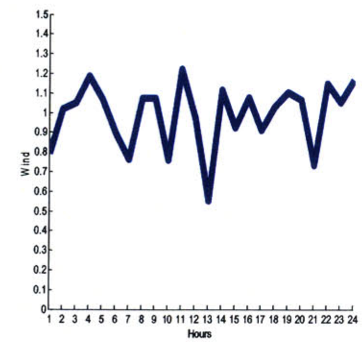
TABLE 4.5.3 The bidding results of the Single Hourly Bidding (SHB) with renewable energy in the HA market

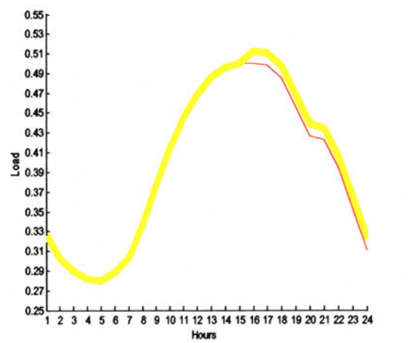
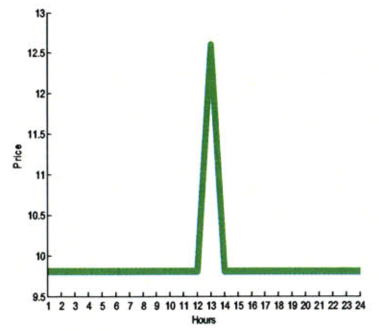
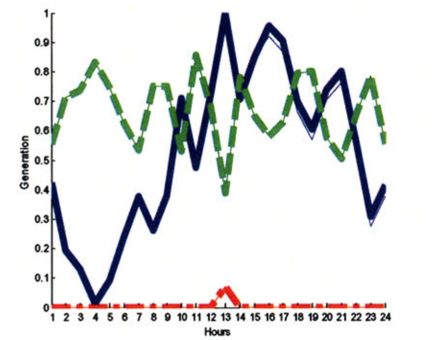
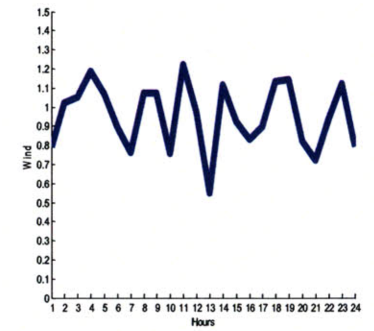
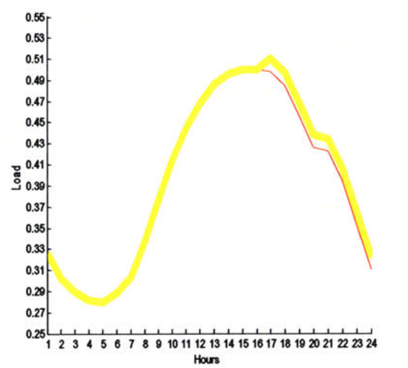
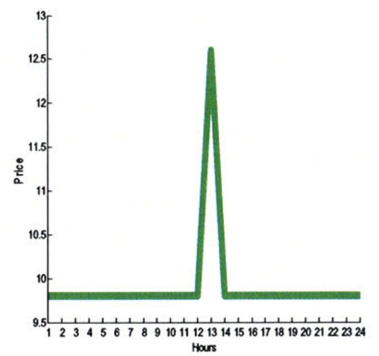
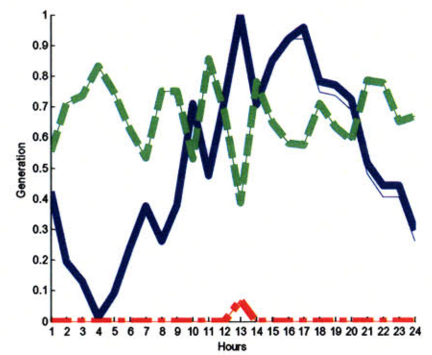
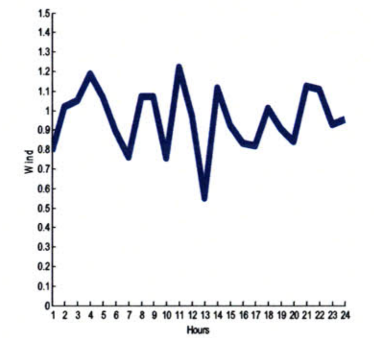
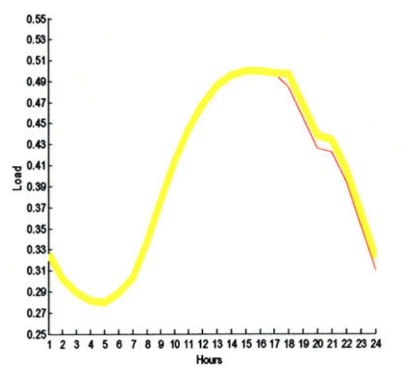
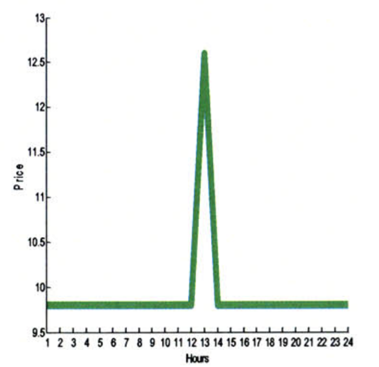
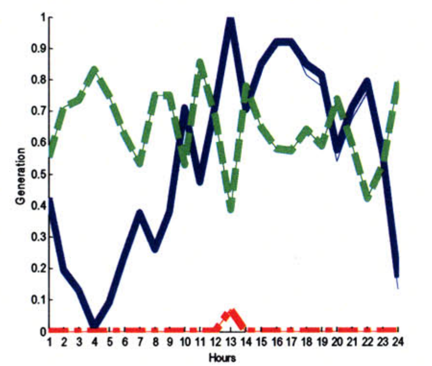
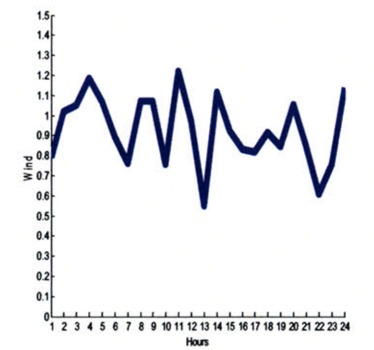
No.	Load Profile	Market Clearing Price	Generation dispatch schedule	Wind Capacity
H1				
H2				
H3				


H4
H5
H6
Load
Price
Generation
Wind
Hours

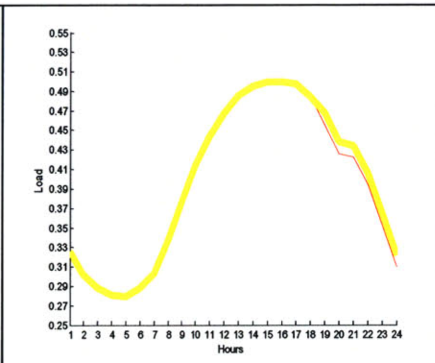
H7	Load Hours	Price Hours	Generation Hours	Wind Hours
H8	Load Hours	Price Hours	Generation Hours	Wind Hours
H9	Load Hours	Price Hours	Generation Hours	Wind Hours

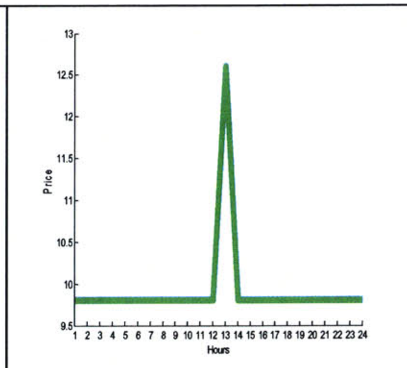
H10				
H11				
H12				

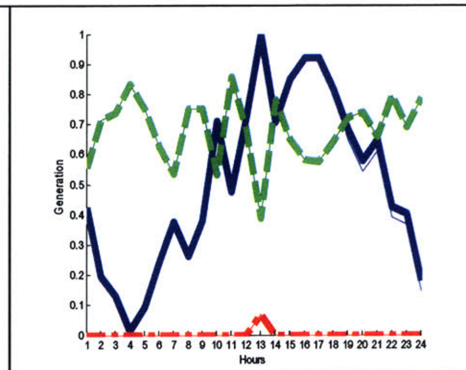
H13	Load Hours	Price Hours	Generation Hours	Wind Hours
H14	Load Hours	Price Hours	Generation Hours	Wind Hours
H15	Load Hours	Price Hours	Generation Hours	Wind Hours

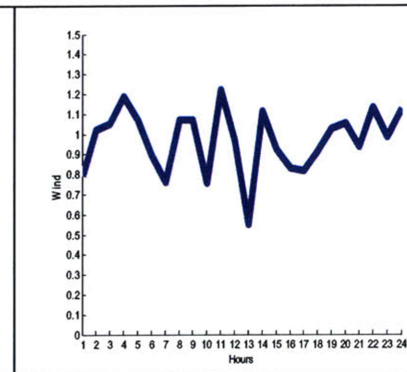
H16	Load Hours	Price Hours	Generation Hours	Wind Hours
H17	Load Hours	Price Hours	Generation Hours	Wind Hours
H18	Load Hours	Price Hours	Generation Hours	Wind Hours

H19

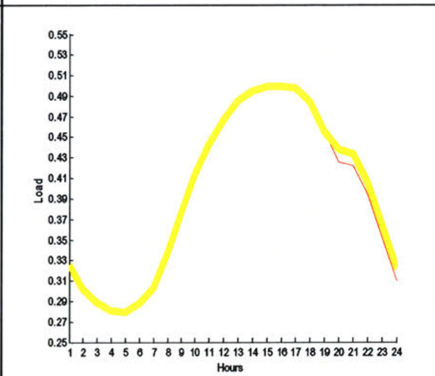

Load
Hours

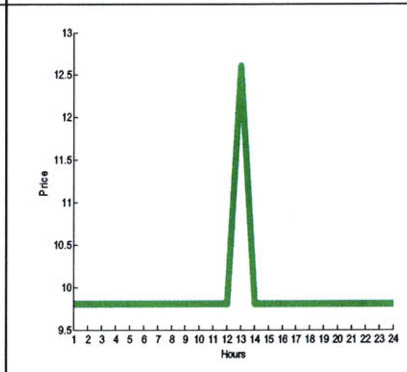

Price
Hours

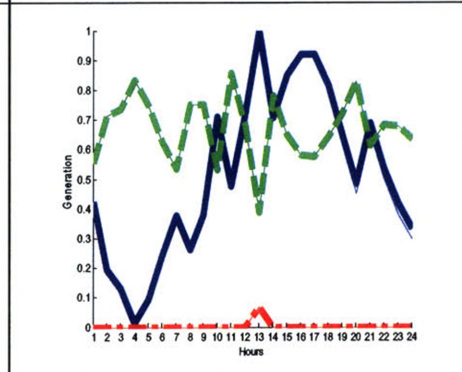

Generation
Hours

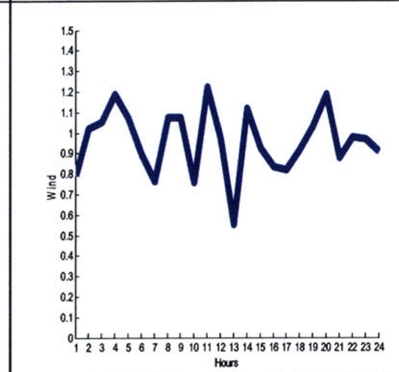

Wind
Hours

H20

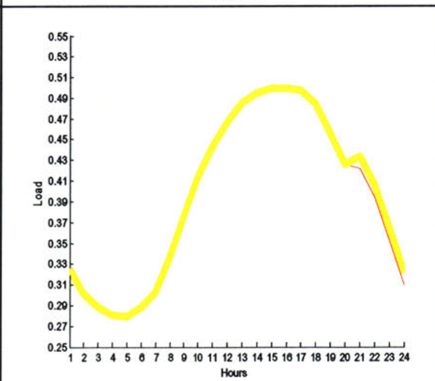

Load
Hours

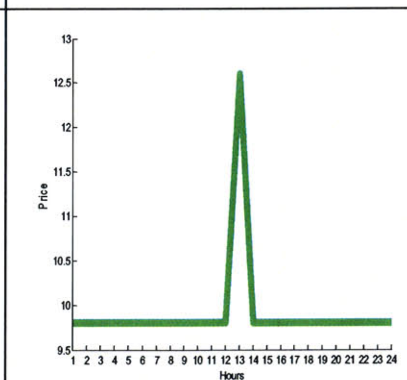

Price
Hours

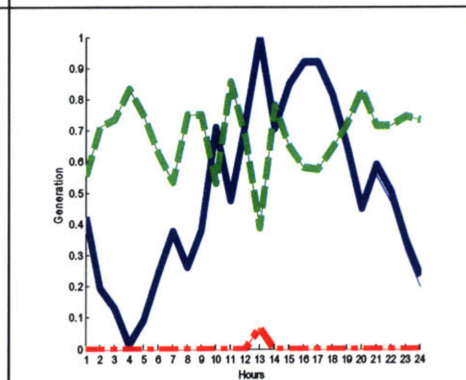

Generation
Hours

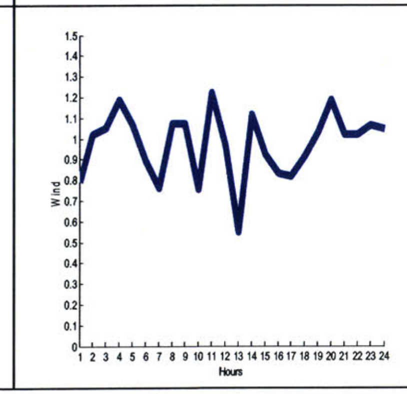

Wind
Hours

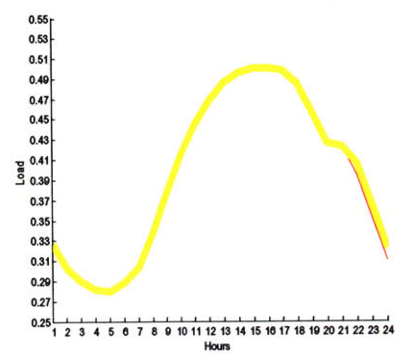
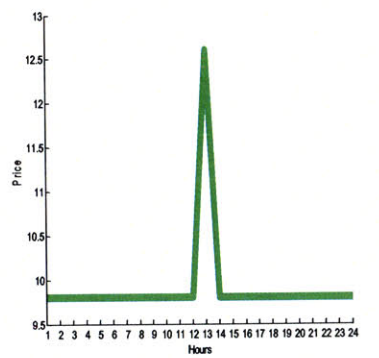
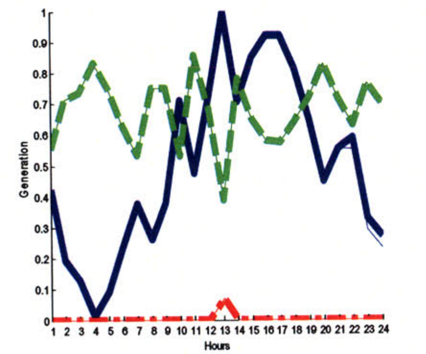
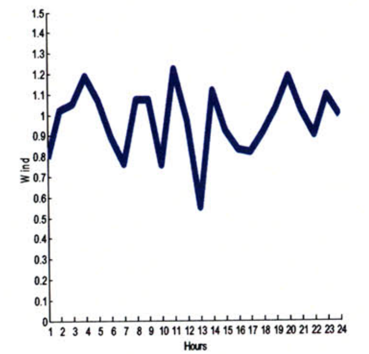
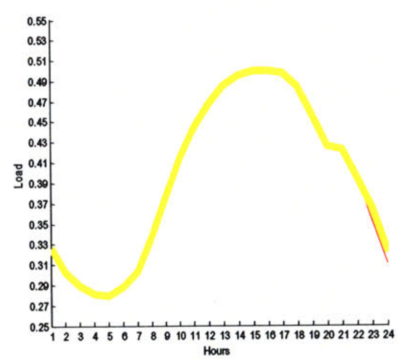
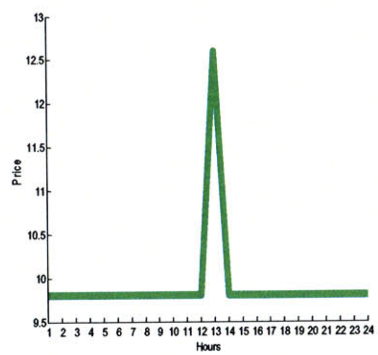
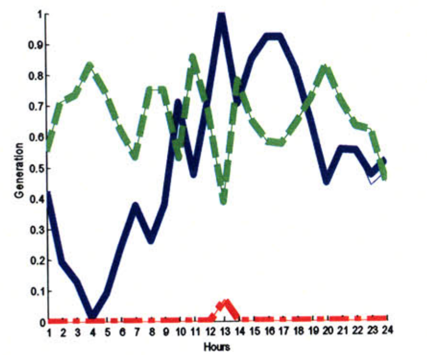
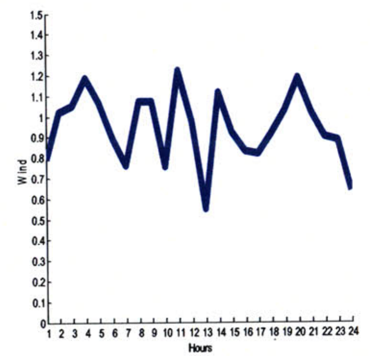
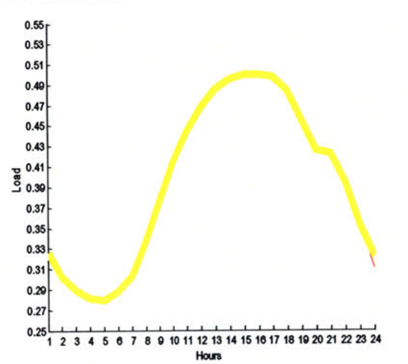
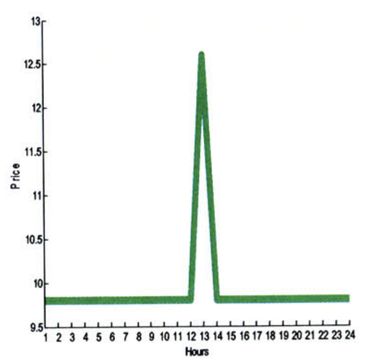
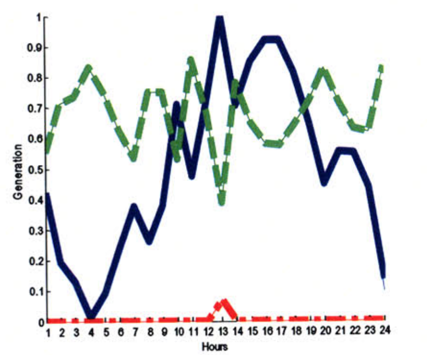
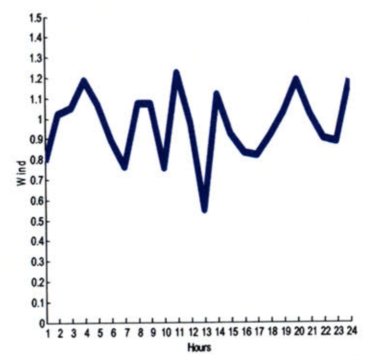
H21


Load
Hours


Price
Hours


Generation
Hours


Wind
Hours

H22				
H23				
H24				

The price settlement rule in the HA market is similar to the two-settlement system introduced in Section 3.3. However, since the HA market is different from the RA market in the sense that it is a forward market, the payment settled in an hourly bidding is not the final payment. This section calculates the hourly payment as:

$$Pay_1 = p_1 \times P_1 \qquad (53a)$$

and

$$Pay_t = Pay_{t-1} + p_t \times (P_t - P_{t-1}), \quad (53b)$$

where Pay_t the payment at Hour t;

p_t the market clearing price at Hour t;

P_t the power bid to be generated or consumed in Hour t according to if Pay_t is the payment to the suppliers or from the retailers.

Therefore, the final payment after the whole of the HA market is:

$Pay_{24} = p_1 \times P_1 + p_2 \times (P_2 - P_1) + \cdots + p_t \times (P_t - P_{t-1}) + \cdots + p_{24} \times (p_{24} - p_{23})$

Figure 4.5.3 and figure 4.5.4 show the final payments from the retailers and to the suppliers respectively based on the above equation. The two figures show that the payment increases when wind capacity is high such as at Hour 5 and Hour 14, see figure 4.5.2. This is because the proposed bidding mechanism encourages electricity consumption when the system has sufficient renewable energy supply by considering the inter-temporal shifting effects. This consumption pattern takes advantage of zero-marginal-cost renewable energy and reduces the RT balancing cost. Moreover, these final payments show that the proposed bidding mechanism reflects the added value of the renewable energy to the system. These benefits are not observed in the SHB, as its final payments to the suppliers and from the retailers shown in figure 4.5.5 and figure 4.5.6.

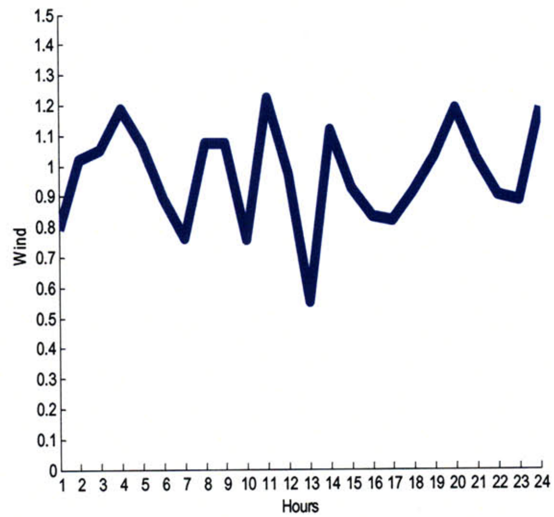


Fig. 4.5.2 The measured wind capacity after the 24 hourly biddings in the HA market

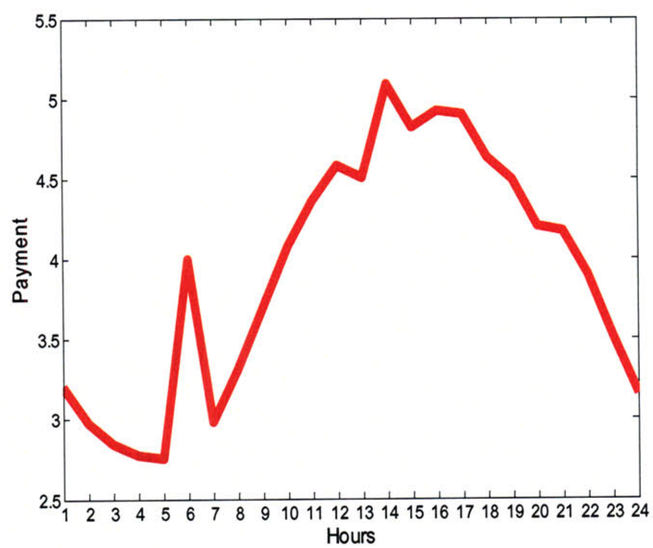


Fig. 4.5.3 The final payment from the retailers in the HA market under the proposed bidding mechanism

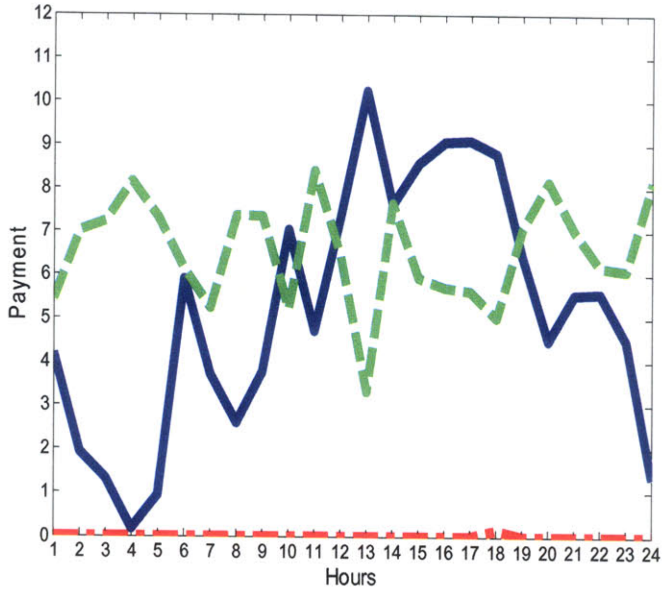


Fig. 4.5.4 The final payment to the suppliers in the HA market under the proposed bidding mechanism

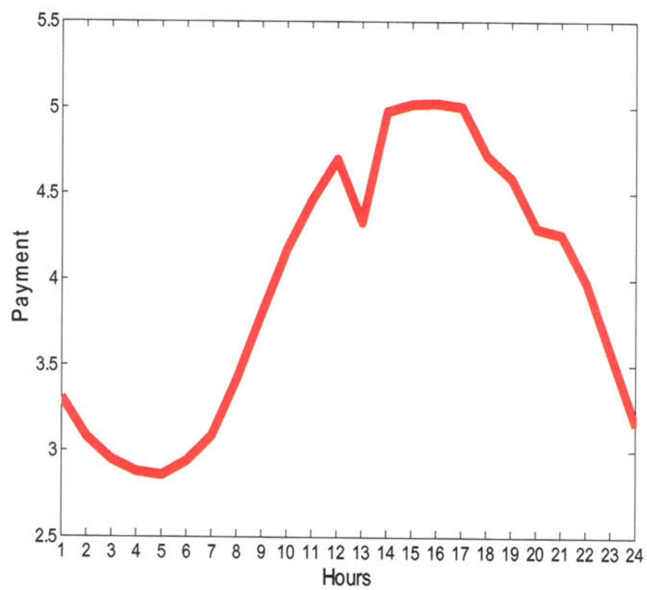


Fig. 4.5.5 The final payment from the retailers in the HA market under the SHB

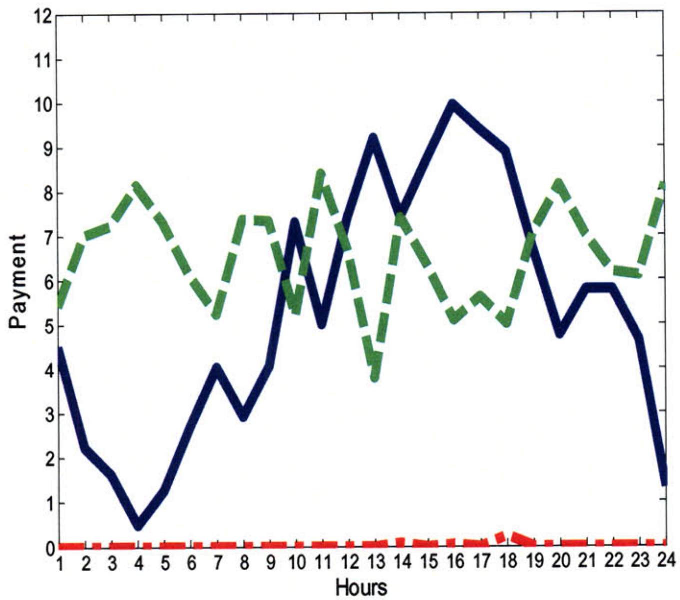


Fig. 4.5.6 The final payment to the suppliers in the HA market under the SHB

4.6 Examples of Non-convergences

This section gives numerical examples of searching for market equilibriums of the non-convergent bidding results of the proposed bidding mechanism. Section 4.2 to Section 4.5 gives numerical examples under various end-user types and system statuses. Some of these examples have results that oscillate between several values and thus are non-convergent. Section 3.4 explains the two causes for the non-convergences: the supply curve's relative slope is steeper than the demand curve's slope and demand clears the market. In addition, Section 3.4 proposes an improved market interaction algorithm to find market equilibriums of these two non-convergent cases. This section gives four numerical examples of the two non-convergent cases. It also shows how the market equilibriums are found following the proposed algorithm.

In the four examples presented in this section, the first two examples show the non-convergences directly caused by demand clearing the market or steep supply curve's relative slope. The other two examples show the non-convergences caused by shifting effects resultant from other non-convergent hours in the timeframe. The four examples are collected from the examples presented in Section 4.2 to Section 4.5. Thus, they have different simulation settings in terms of end-user types, system statuses and sub-market types. The simulation setting details of these examples will not be repeated in this section. In addition, for analysis convenience, only the load profile and the market clearing price of the bidding results are presented in this section. Detailed simulation settings and bidding results of the four examples can be found in their first presentations if interested.

Non-convergence caused by demand clears the market

Figure 4.5.1 and figure 4.5.2 give the example of non-convergence caused by demand clearing the market at a single hour. This example is from the Hour 10's bidding results in Section 4.5, and thus we only need to find the market equilibrium for the hours after Hour 10.

Figure 4.5.2 shows that due to the lack of wind generation capacity, the market clearing price at Hour 12 oscillates between the marginal cost of G3 (12.6) and the marginal cost of G1 (9.8). This price oscillation results in load profile oscillation, shown in figure 4.5.1. Since G2 is a wind generation unit in the system and has a marginal cost as 0.0 at all the hours, the bidding results' non-convergence is caused by demand clearing the market based on the algorithm in figure 3.3.11. We follow the demand curve searching algorithm in Section 3.4.2 to find the market equilibrium of this example:

1. Denote Hour 12 as the hour H_{CDM} when demand clears the market; Denote all the other hours in the timeframe as H_0;
2. Set p_{12} as variable and d_{12} as $1/3$, thus the total demand of the three retailers at Hour 12 equals the capacity of G1 as 1.0; Set the price at H_0 as their convergent value 9.8 and the demand at H_0 as variables;
3. Substitute all parameters and variables into equation (15), where the PEM is the of real-world end-user type;
4. By solving equation (15) in step 3, we obtain the demand and price at all the hours. Because the obtained $p_{12} \in (9.8, 12.6)$ and the demand at H_0 is less than $1/3$, the solution is accepted.

The obtained demand and price are plotted in figure 4.5.3 and figure 4.5.4. Figure 4.5.4 shows that the curtailing premium equals to $p_{12} - 9.8$, which prevents end users consuming more electricity and increasing the system's marginal cost to 12.6. Notice that the demand and price before Hour 10 are derived from bidding results from the previous HA markets.

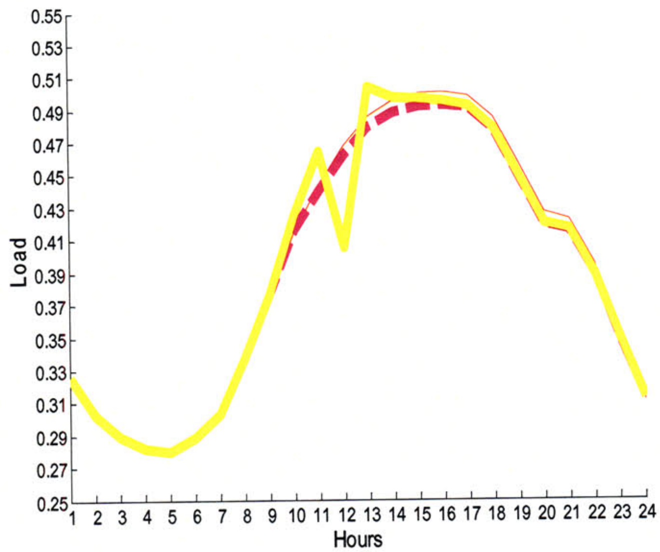


Fig. 4.5.1: The non-convergent load profile caused by demand clears the market at a single hour.

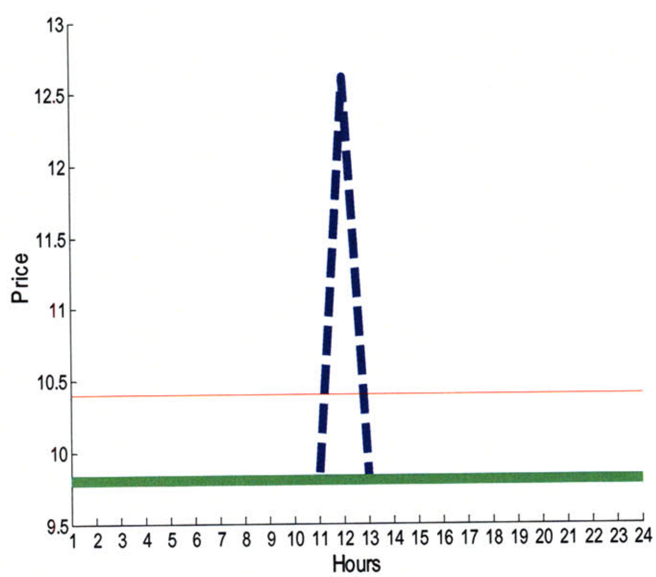


Fig. 4.5.2: The non-convergent market clearing price caused by demand clears the market at a single hour

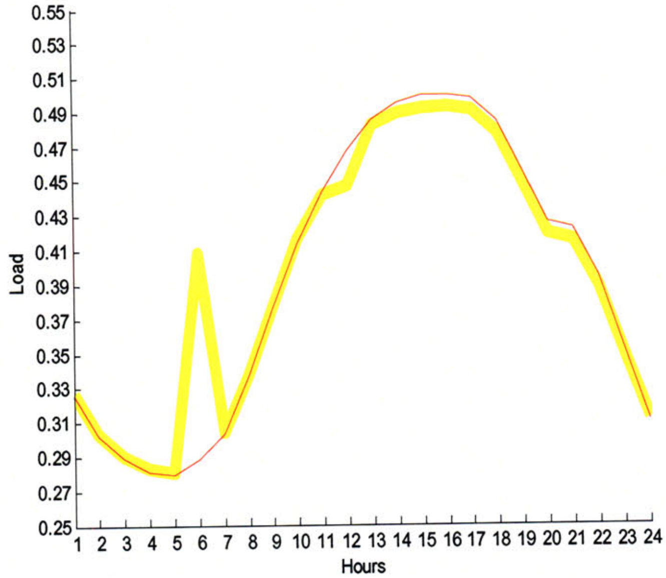


Fig. 4.5.3: The load profile at the market equilibrium. The market equilibrium is derived from the non-convergent bidding result caused by demand clears the market at a single hour.

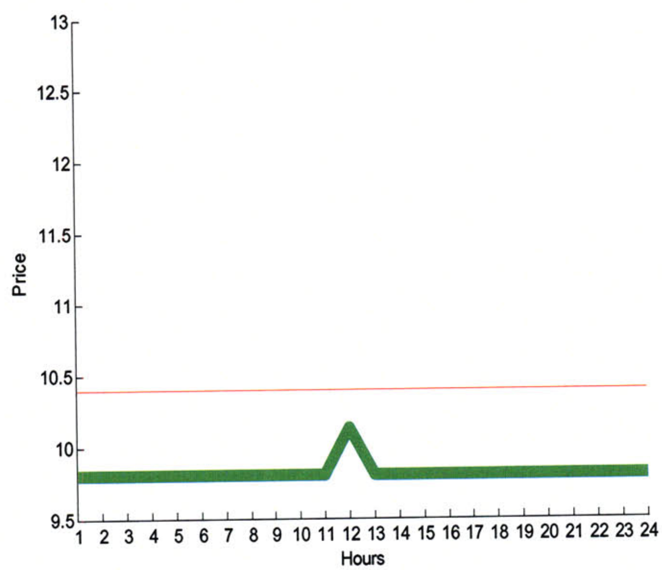


Fig. 4.5.4: The market clearing price at the market equilibrium. The market equilibrium is derived from the non-convergent bidding result caused by demand clears the market at a single hour.

Non-convergence caused by steep supply curve's relative slope

Figure 4.5.5 and figure 4.5.6 give the example of non-convergence caused by demand clearing the market at a single hour and steep supply curve's at all the other hours. This example is from the bidding results when the system loses G2's capacity at Hour 2, in Section 4.3.

Figure 4.5.6 shows that the market clearing prices oscillates between G1's marginal cost (9.8) and G3's marginal cost (12.6). This price oscillation results in load profile oscillation, shown in figure 4.5.5. At Hour 2, the system loses G2's capacity, and the only two available generation units are G1 and G3. Therefore, the algorithm detects the price oscillation at this hour is caused by demand clearing the market. However, the system has all the three generation units available at all the other hours. Based on the improved market interaction algorithm, the price oscillations at all these hours are caused by the steep supply curve's relative slope. Therefore, the algorithm resets the market clearing prices at all the hours except Hour 2 to 10.7, which is the marginal cost between 9.8 and 12.6 in the system. Afterwards, it searches on the demand curve for the market equilibrium at Hour 2 according to the algorithm in Section 3.4.2:

1. Denote Hour 2 as the hour $H_{CDM}$when demand clears the market; Denote all the other hours in the timeframe as $H_{\widetilde{DCM}}$;
2. Set p_2 as variable and d_2 as $1/3$, thus the total demand of the three retailers at Hour 2 equals the capacity of G1 as 1.0; Set the price at H_0 as the reset value 10.7 and the demand at H_0 as variables;
3. Substitute all parameters and variables into equation (15), where the PEM is the of real-world end-user type;
4. By solving equation (15) in step 3, we obtain the demand and price at all the hours. The obtained $p_{12} \in (9.8, 12.6)$, and the demand at $H_{\widetilde{DCM}}$ are in$(0.33, 0.57)$, which means G2 is the marginal unit at all the hours except at Hour 2. Therefore, the solution is accepted.

The obtained demand and price are plotted in figure 4.5.7 and figure 4.5.8.

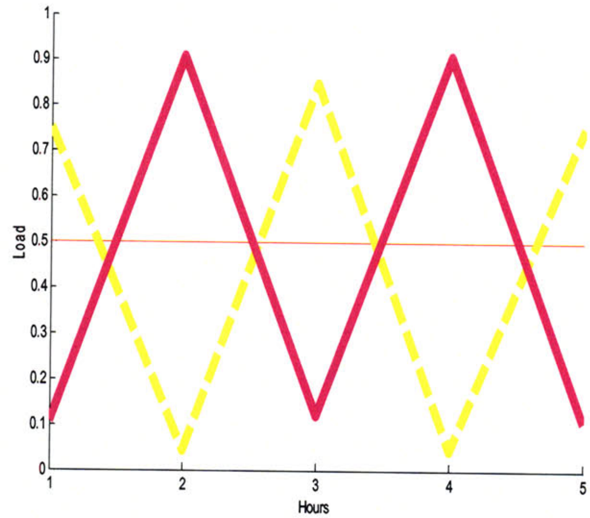


Fig. 4.5.5 The non-convergent load profile caused by steep local relative slope of the supply curve.

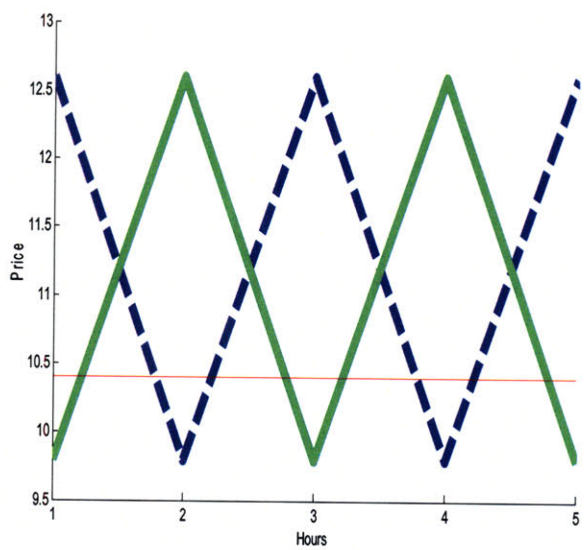


Fig. 4.5.6: The non-convergent market clearing price caused by steep local relative slope of the supply curve.

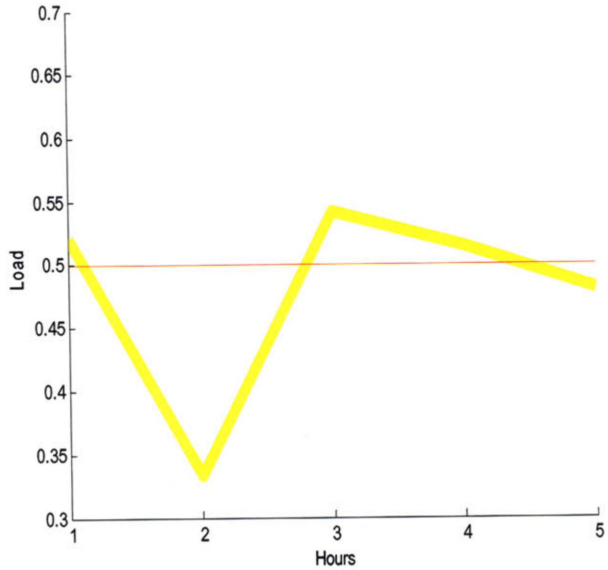


Fig. 4.5.7 The load profile of the market equilibrium. The market equilibrium is derived from figure 4.5.5 and figure 4.5.6

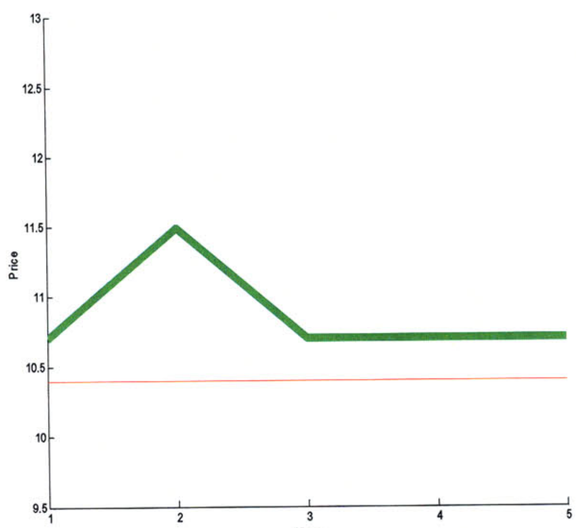


Fig. 4.5.8 The market clearing price of the market equilibrium. The market equilibrium is derived from figure 4.5.5 and figure 4.5.6

Non-convergence caused by demand clears the market and shifting effects

Figure 4.5.9 and figure 4.5.10 give the example of non-convergence caused by demand clearing the market and shifting effects. This example is from the Hour 13's bidding results in Section 4.5, and thus we only need to find the market equilibrium for the hours after Hour 13.

Figure 4.5.10 shows that due to the lack of wind generation capacity, the market clearing prices at Hour 13 and Hour 14 oscillate between the marginal cost of G3 (12.6) and the marginal cost of G1 (9.8). This price oscillation results in load profile oscillation, shown in figure 4.5.11. Since G2 is a wind generation unit in the system and has a marginal cost as 0.0 at all the hours, the bidding results' non-convergence is caused by demand clearing the market based on the algorithm in figure 3.3.11. However, the price oscillation at Hour 13 can be caused by demand clearing the market at that hour, or it can be caused by the shifting effects from Hour 14 when the demand clears the market. The same statement applies to the price oscillation at Hour 14 as well. Thus, we need to consider all these non-convergence causes when search for the market equilibrium on the demand curve:

1. Consider three possible causes for the non-convergence: demand clears the market at Hour 13 and at Hour 14, demand clears the market at Hour 13 and shifting effects from Hour 13 cause oscillation at Hour 14, and demand clears the market at Hour 14 and shifting effects from Hour 14 cause oscillation at Hour 13.
2. According to the three possible causes, denote Hour 13 or Hour 14 as the hour H_{CDM} when demand clears the market or $H_{\widetilde{DCM}}$ when non-convergence is caused by shifting effects; Denote all the other hours in the timeframe as H_0;
3. Set p_{DCM} as variable and H_{DCM} as $1/3$, thus the total demand of the three retailers at H_{DCM} equals the capacity of G1 as 1.0; Set the price at $H_{\widetilde{DCM}}$ as their oscillating value 9.8 or 10.7 and the demand at $H_{\widetilde{DCM}}$ as variables; Set the price at H_0 as their convergent value 9.8 and the demand at H_0 as variables;

4. Substitute all parameters and variables into equation (15), where the PEM is the of real-world end-user type;
5. By solving equation (15) in step 3, we obtain the demand and price at all the hours. Check if the obtained $p_{DCM} \in (9.8, 12.6)$ and the demand at $H_{\overline{DCM}}$ is less than $1/3$. The solution is accepted, when the condition is satisfied.

After trying all the three possible causes, the solution is feasible when demand clears the market only at Hour 13. The price oscillation at Hour 14 is caused by the shifting effects from the non-convergence at Hour 13. The obtained demand and price are plotted in figure 4.5.11 and figure 4.5.12. Figure 4.5.12 shows that the curtailing premium equals to $p_{13} - 9.8$, which prevents end users consuming more electricity and increasing the system's marginal cost to 12.6. Notice that the demand and price before Hour 12 are derived from bidding results from the previous HA markets.

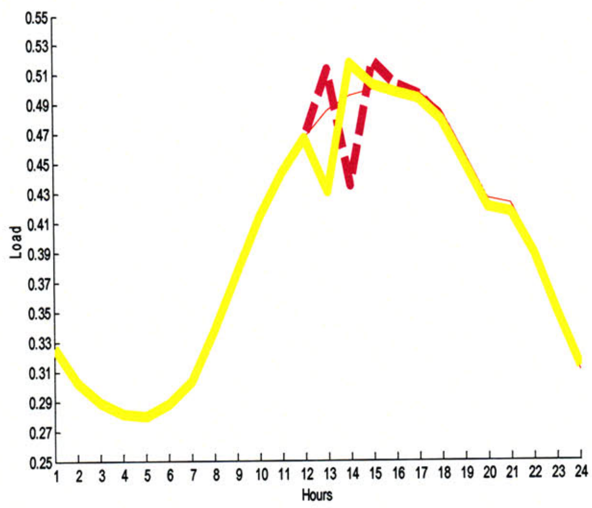


Fig. 4.5.9: The non-convergent load profile caused by demand clears the market and shifting effects.

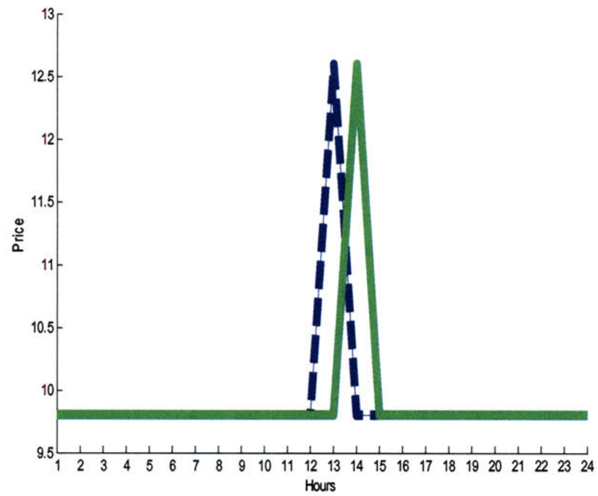


Fig. 4.5.10: The non-convergent market clearing price caused by demand clears the market and shifting effects.

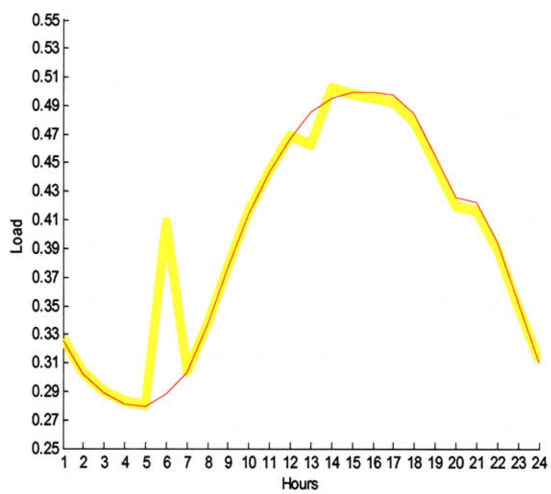


Fig. 4.5.11: The load profile of the market equilibrium. The market equilibrium is derived from figure 4.5.9 and figure 4.5.10.

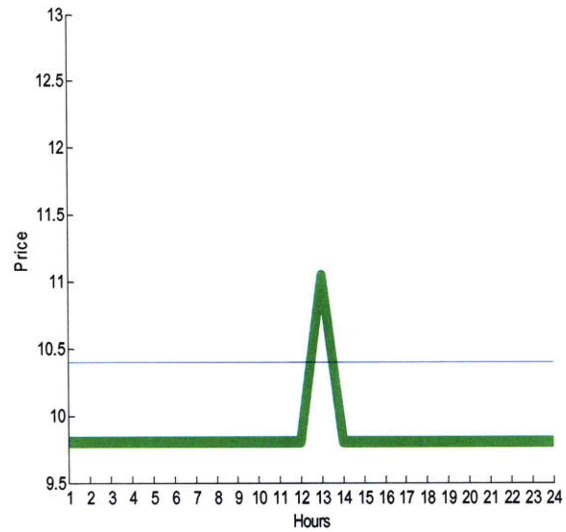


Fig. 4.5.12 : The market clearing price of the market equilibrium. The market equilibrium is derived from figure 4.5.9 and figure 4.5.10.

Non-convergence caused by demand clears the market at multiple hours

Figure 4.5.13 and figure 4.5.14 give the example of non-convergence caused by demand clearing the market at multiple hours. This example is from the Case 2's bidding results in Section 4.3.

Figure 4.5.14 the market clearing prices at Hour 2, Hour 7 and Hour 24 oscillate between the marginal cost of G2 (10.7) and the marginal cost of G1 (9.8). This price oscillation results in load profile oscillation, shown in figure 4.5.13. Since G1 and G2 are two generation units that have the marginal cost next to each other, the bidding results' non-convergence is caused by demand clearing the market based on the algorithm in figure 3.3.11. However, for each hour when the price oscillates, the non-convergence can be caused by demand clears the market at that hour or by shifting effects from other hours when demand clears the market. Based on the first two steps of the demand curve searching algorithm in Section 3.4.2, all the parameter settings of Hour 2, Hour 7 and Hour 24 under all the non-convergence's causes are listed in Table 4.4.1:

TABLE 4.4.1 Parameters of Hour 2, Hour 7 and Hour 19 (H2, H7, H19)

No.	Load	Price	No.	Load	Price
1	(0.33,0.33,0.33)	(v, v, v)	2	(0.33, 0.33, v)	(v, v, 9.8)
3	(0.33,0.33, v)	(v, v, 10.7)	4	(0.33, v, 0.33)	(v, 9.8, v)
5	(0.33, v, 0.33)	(v, 10.7, v)	6	(v, 0.33, 0.33)	(9.8, v, v)
7	(v, 0.33, 0.33)	(10.7, v, v)	8	(v, v, 0.33)	(9.8, 10.7, v)
9	(v, 0.33, v)	(9.8, v, 9.8)	10	(0.33, v, v)	(v, 10.7, 9.8)
11	(v, v, 0.33)	(10.7, 9.8, v)	12	(v, 0.33, v)	(10.7, v, 10.7)
13	(0.33, v, v)	(v, 9.8, 10.7)	14	(v, v, 0.33)	(9.8, 9.8, v)
15	(0.33, v, v)	(v, 9.8, 9.8)	16	(0.33, v, v)	(v, 10.7, 10.7)
17	(v, v, 0.33)	(10.7, 10.7, v)	18	(v, 0.33, v)	(10.7, v, 9.8)
19	(v, 0.33, v)	(10.7, v, 9.8)			

*v denotes the variable to be decided.

For each non-convergence causes listed in Table 4.4.1, substitute its parameter settings into equation (15), where the PEMs' types are the early end users, forward shifting end users and real-world end users. By solving equation (15) in step 3, we obtain the demand and price at all the hours. Check (1) if the obtained $p_{DCM} \in (9.8, 10.7)$ and (2) $d_{\widetilde{DCM}} \leq 1/3$ if $p_{\widetilde{DCM}} = 9.8$ or $d_{\widetilde{DCM}} \in (0.33, 0.56)$ if $p_{\widetilde{DCM}} = 10.7$. The solution is accepted, when the condition is satisfied.

After trying all the three possible causes, the solution is feasible when demand clears the market at all the three hours H2, H7 and H24. The obtained demand and price are plotted in figure 4.5.15 and figure 4.5.16. Figure 4.5.15 shows that the curtailing premium equals to $p_{DCM} - 9.8$, which prevents end users consuming more electricity and increasing the system's marginal cost to 10.7.

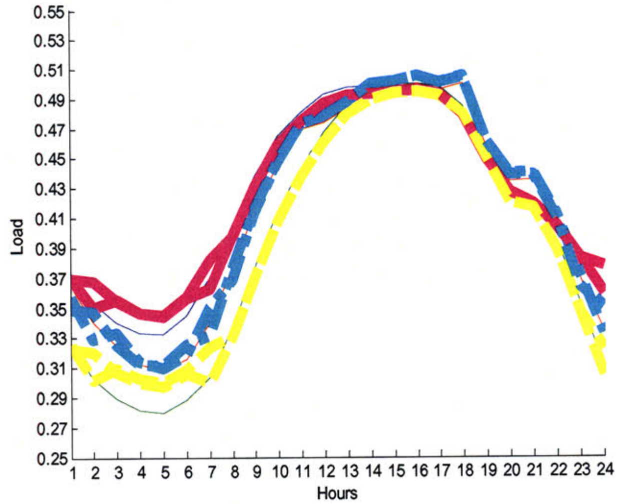


Fig. 4.5.13: The non-convergent load profile caused by demand clears the market at multiple hours.

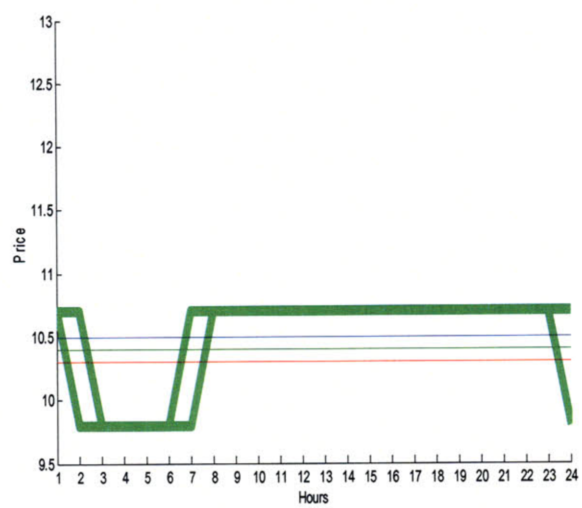


Fig. 4.5.14: The non-convergent market clearing price caused by demand clears the market at multiple hours.

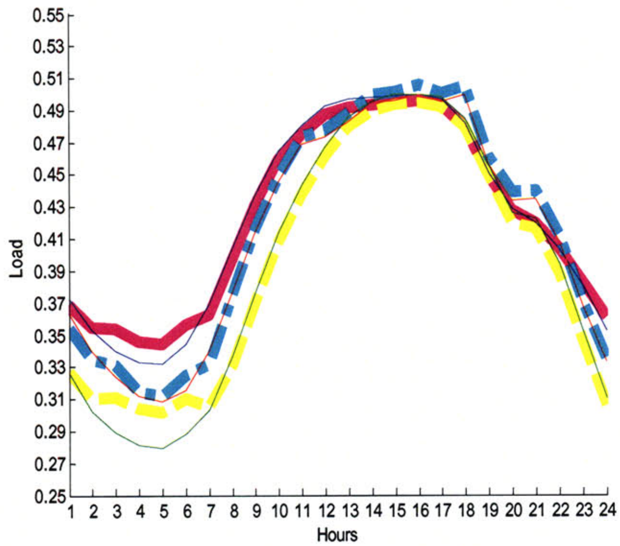


Fig. 4.5.15 The load profile of the market equilibrium derived from figure 4.5.13 and figure 4.5.14.

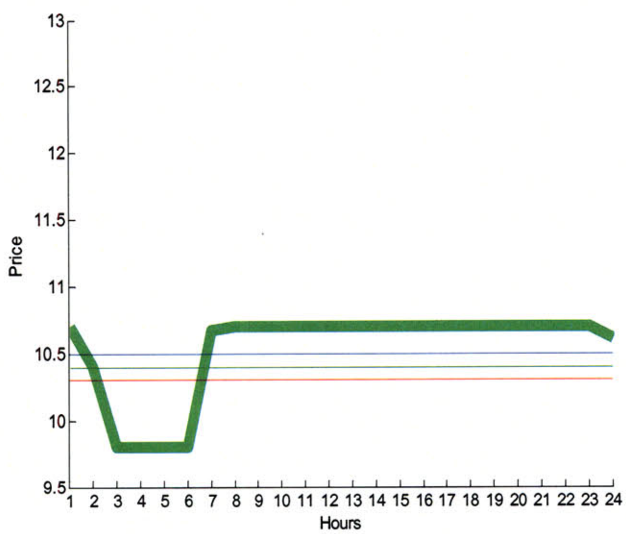


Fig. 4.5.16: The market clearing price of the market equilibrium derived from figure 4.5.13 and figure 4.5.14.

Chapter 5

Conclusion

In this thesis, we propose a demand responsive bidding mechanism which considers end-user response with inter-temporal load shifting in wholesale electricity pools. Bidding rules, bidding acceptance rules and settlement rules are defined for this bidding mechanism. PEMs are used to model all end-user response types. Bidding results obtained from the proposed bidding mechanism are closer to the actual market equilibrium.

Mathematical models of the proposed bidding mechanism are formulated in an optimization problem. By deriving the closed-form solution of this problem, we show that the pricing structure of the proposed bidding mechanism is the same as the spot pricing structure proposed in Schweppe's work. In addition, we show that the shadow prices of the generation and demand inter-temporal conditions are included in the market clearing price under the proposed bidding mechanism.

Sensitivity analysis of the proposed bidding mechanism are conducted under four disturbances: changing in generation cost, changing in generation capacity, changing in transmission limits and changing in demand response programs. The condition of the same bidding acceptance under these four disturbance types are derived.

In addition, we give a full classification of the PEMs based on end-user response types. Furthermore, we point out the factors affecting the PEM's establishment: affecting periods, affected periods and incentive timings of demand response programs. We also present several methods of estimating the PEMs.

To implement the proposed bidding mechanism, we develop an improved algorithm based on the market interaction algorithm in David's work. This improved algorithm can detect the two causes for market non-convergence: demand clears the market and steep local relative slope of the supply curve, where the second condition and the concept of *relative slope* is defined originally in this work. By applying this improved algorithm, market equilibriums are found in the previous non-convergent cases.

The proposed bidding mechanism's advantages over the traditional bidding mechanisms are shown by numerical examples in DA and HA markets. It shows that the bidding results of the proposed bidding mechanism are closer to the actual market equilibriums. For this reason, applying this bidding mechanism can avoid problems such as low market efficiency, high RT market balancing cost and low reliability.

Moreover, under contingencies, the proposed bidding mechanism guarantees sufficient end user electric use by considering inter-temporal load shifting to ensure the end-user utility level. In addition, it reduces the price spike most compared to the two traditional bidding mechanisms.

In systems with renewable energy, the proposed bidding mechanism encourages electricity consumption when the system has sufficient capacities and discourages the electricity consumption by giving proper market clearing price. The resultant consumption pattern takes advantage of zero-marginal-cost renewable energy and reduces the RT balancing cost. In addition, the final payments obtained under the proposed bidding mechanism reflect the added value of the renewable energy to the system.

Appendix A

Numerical Examples' Case Description

TABLE 1: Reference Case

Generation	Non-renewable power plants Upper capacity constraint $0 \leq Y_j(t) \leq K_j$
Transmission	No constraint
PEM	Only with self-elasticity | Single Hourly Bid
Contingency	No contingency
Market type	DA market
Market equilibrium	Generation clears the market

TABLE 2: Categories Description

Generation	a. non-renewable plants b. renewable plants *Constraints:* c. only upper capacity constraint $0 \leq Y_j(t) \leq K_j$ d. positive lower capacity $\underline{K}_j \leq Y_j(t) \leq K_j$ and $\underline{K}_j > 0$ e. ramp-up and ramp-down constraints f. least on/offline time
Transmission	a. no network constraints b. power flow constraints of transmission lines c. current flow constraints of transmission lines d. voltage level constraints of buses e. phase difference constraints of buses
PEM	a. curtailable load b. early end users c. late end users d. forward shifting end users e. backward shifting end users f. flexible end users g. real-world end users h. distributed generation i. on-site storage
Contingency	a. no contingency b. sudden increment in one generation unit's cost c. loss of a transmission line in a certain hour d. loss of a generation unit in a certain hour
Market type	a. DA market only b. DA and RT market in sequence
Market equilibrium	a. generation clears the market b. demand clears the market

TABLE 3: Simulated Cases

Case Categories	No.	Generation	Transmission	PEM	Contingencies	Market	Market Equilibrium
Reference case	1	a,c	a	a	a	a	a
PEM variations	2	a,c	a	b	a	a	a
	3	a,c	a	c	a	a	a
	4	a,c	a	d	a	a	a
	5	a,c	a	e	a	a	a
	6	a,c	a	f	a	a	a
	7	a,c	a	g	a	a	a
	8	a,c	a	h	a	a	a
	9	a,c	a	i	a	a	a
Demand clearing market	10	a,c	a	g	a	a	b
Contingencies	11	a,c	a	g	b	a	a
	12	a,c	a	g	c	a	a
	13	a,c	a	g	d	a	a
Renewable energy	14	b,c	a	g	a	b	a
Comprehensive case	15	b,d,e	b	h,d,g	b	b	a

Appendix B

Code of the Numerical Examples

Generating 24 × 24 *PEMs for the Ten End-User Types*

```
% PEM generation 24 X 24
% self-elasticity   0.02
% PEM0    curtailable end users               PEM1     early end users
% PEM2    late end users                      PEM3     forward shifting
end users
% PEM4    backward shifting end users         PEM5     flexible end
users
% PEM6    real world end users


PEM0 = -0.02.*eye(24);


for i = 1:24
% PEM1
   if i <= 7
       PEM1(:,i) = 0.02/6.*[ones(7,1);zeros(17,1)];
   else
       PEM1(:,i) = 0.02/7.*[ones(7,1);zeros(17,1)];
   end
       PEM1(i,i) = -0.02;
% PEM2
   if i >= 17
       PEM2(:,i) = 0.02/6.*[zeros(17,1);ones(7,1)];
   else
       PEM2(:,i) = 0.02/7.*[zeros(17,1);ones(7,1)];
   end
       PEM2(i,i) = -0.02;
% PEM3
    if i == 24
        PEM3(:,i)= zeros(24,1);
    else
        PEM3(:,i) = (0.02/(24-i)).*[zeros(i,1);ones(24-i,1)];
    end
    PEM3(i,i) = -0.02;
% PEM4
    if i == 1
        PEM4(:,i) = zeros(24,1);
    else
        PEM4(:,i) = (0.02/(i-1)).*[ones(i,1);zeros(24-i,1)];
    end
    PEM4(i,i) = -0.02;
```

```
% PEM5
    PEM5(:,i) = 0.02/23.*ones(24,1);
    PEM5(i,i) = -0.02;
% PEM6
    for j = 1:24
        if j == i
            PEM6(j,i) = -0.02;
        else
            PEM6(j,i) = 0.02/((abs(i-j)*1.1)^2+1);
        end
    end
end

% using weibull function to generate wind coefficient. * Every running
of
% 'random' function produces different results.
% WindCo = ones(24,24);
% for i = 1:24
%     WindCo(:,i) = random('wbl',1.*ones(24,1),7.*ones(24,1));
%     if i > 1
%         WindCo(1:i-1,i) = WindCo(1:i-1,i-1);
%     end
% end

load('THS_windco.mat','WindCo','-mat')
```

Running the Proposed Bidding Mechanism in the DA Market with the Five-Period Data Set

```
function IAS09_0310_b_nwr
emgen_THS24
% PEM[10] = Emload2;
% PEM[37] = Emload8;
% PEM[14]= Emload4;

PEM[8]= -0.2.*eye(5);
PEM{2} = PEM{1};
PEM{3} = PEM{1};
loop  = 10;

Pd = zeros(5,3);
Qd = zeros(5,3);
% Qref =  [0.089 0.0456 0.0559;
%          0.3329 0.3671 0.0897;
%          0.2231 0.6802 0.0499;
%          0.2326 0.6655 0.048;
%          0.3078 0.7959 0.0474];
%Pref = [11.2*ones(5,1) 10.5*ones(5,1) 13.2*ones(5,1)];

%===ref 0304 value ===
%Pref = [11.6 9.8 10.7 14.5 12.6];
%Qorg = [0.0989 0.0456 0.0799;0.2497 0.3059 0.0523;0.2479 0.6802
0.0712;0.2326 0.6655 0.048;0.2565 0.6632 0.0474];
%====================
```

```
%===ref THS 1 value===
Pref1 =  ones(5,1)*10.5; %RT H5
Pref2 = ones(5,1)*10.4;
Pref3 = ones(5,1)*10.3;
Pref = [Pref2 Pref2 Pref2];
%Qref = [0.554.*ones(1,3);0.4967.*ones(1,3); 0.531712.*ones(1,3);
%0.527694.*ones(1,3);0.557541.*ones(1,3)]; %RT H5
Qref1 = ones(5,1)*0.57;
Qref2 = ones(5,1)*0.5;
Qref3 = ones(5,1)*0.42;
Qref = [Qref2 Qref2 Qref2];

%====================

%Qmin = [0.9 1 0.7];
%Qmax = [1 1 1];

Qorg = Qref;

for kk = 1:loop
    if kk ==1
        Ch = Qorg;
        read_in(Ch);
        [ps,mcp] = call_ed;
%RT market
%        mcp = [mcp(1);9.8;10.1279;9.8;9.8;mcp(end)];
        [nhr,nsup] = size(ps);
    else
        read_in(Ch); %update the datafile
        [ps,mcp] = call_ed;
%RT market
%        mcp = [mcp(1);9.8;10.1279;9.8;9.8;mcp(end)];
    end
    ps_rec{kk}  = ps;
    ps = ps(2:nhr,:);
    mcp_rec{kk} = mcp;
    mcp = mcp(2:nhr,:);
    for i = 1:3
        Pd(:,i) = mcp - Pref(:,i);
        Qd(:,i) = PEM{i} * Pd(:,i);
        Q(:,i)  = Qd(:,i) + Qref(:,i);
 %       Q1(:,i) = Qorg(:,i).*Qmin(i);
 %       Q2(:,i) = Qorg(:,i).*Qmax(i);
    end
%    Q = max(Q, Q1);
%    Q = min(Q, Q2);
    Ch = Q;
    ch_rec{kk+1} = Q;
end
    ch_rec{1} = Qref;

for ll = 1:loop
    if ll ==1
    figure
    h1 = axes;
```

```
    hold on
    plot(ps_rec{1}(2:nhr,:))
    figure
    h2 = axes;
    hold on
    plot(mcp_rec{1}(2:nhr,:))
    figure
    h3 = axes;
    hold on
    plot(ch_rec{1}(1:nhr-1,:))
    else
%         plot(h1,ps_rec{ll}(2:nhr,:),'LineWidth',6)
        plot(h1,ps_rec{ll}(2:nhr,1),'-','LineWidth',6,'Color','blue')
        plot(h1,ps_rec{ll}(2:nhr,2),'--','LineWidth',6,'Color','green')
         plot(h1,ps_rec{ll}(2:nhr,3),'-.','LineWidth',6,'Color','red')
        plot(h2,mcp_rec{ll}(2:nhr,:),'LineWidth',6,'Color','green')
        plot(h3,ch_rec{ll}(1:nhr-1,1),'-
','LineWidth',6,'Color','magenta')
         plot(h3,ch_rec{ll}(1:nhr-1,2),'--
','LineWidth',6,'Color','yellow')
          plot(h3,ch_rec{ll}(1:nhr-1,3),'-
.','LineWidth',6,'Color','cyan')
    end
end
disp('end of loop')
saveas(h1,'J:\MS Thesis\Thesis\Chapter 4_figures\13_G_SHB.fig')
saveas(h2,'J:\MS Thesis\Thesis\Chapter 4_figures\13_p_SHB.fig')
saveas(h3,'J:\MS Thesis\Thesis\Chapter 4_figures\13_D_SHB.fig')
fclose('all')
%% input interface
function read_in(Ch)

%write into a temporary file
fid00 = fopen('psatdata_THS_11.gms','r'); %initialize
fid11 = fopen('psatdata.gms','w+'); %create a temporary file
frewind(fid00);
while 1
    t00 = fgetl(fid00);
%    t11 = fgetl(fid11);
    if strcmp(t00,'$kill Ch'),
        break
    else
       fprintf(fid11,'%s\n',t00);
    end
end
%t11 = fgetl(fid11); %point to the next line of 'parameter Ch'
fprintf(fid11,'$kill %s\n','Ch');
fprintf(fid11,'parameter %s /\n','Ch');
[nH,nLd] = size(Ch);
for j = 1:nLd,
        fprintf(fid11,'%s%s.%d %f\n','H','0',j,Ch(1,j));
    for i = 1:nH,
        fprintf(fid11,'%s%d.%d %f\n','H',i,j,Ch(i,j));
%        t11 = fgetl(fid00);
    end
end
fprintf(fid11,'/;\n');
```

```
fprintf(fid11,'%s\n','$offempty');
%write back
fid00 = fopen('psatdata_THS_11.gms','w+');
fid11 = fopen('psatdata.gms','r'); %create a temporary file
frewind(fid11);
while 1
    t11 = fgetl(fid11);
    if strcmp(t11,'$offempty');%0429
        break
    else
        fprintf(fid00,'%s\n',t11);
    end
end
fprintf(fid00,'%s\n','$offempty');
fclose(fid11);
fclose(fid00);

%% ---------------------------------------------------------------
function varargout = call_ed
    status = 0;
    t0 = clock;
    [status,result] = system(['gams ','fm_THS_1.gms']);
    disp([' GAMS routine completed in ',num2str(etime(clock,t0)),' s'])
    if status
        disp(result)
        disp('Error!!')
    return
    end
    nout = 0;
    EPS = eps;
    clear psatsol
    psatsol
    if nout < nargout
        for i = nout+1:nargout
            varargout{i} = [];
        end
    end

    if nout > nargout
        varargout(nargout+1:nout) = [];
    end
```

Running the Proposed Bidding Mechanism in the DA Market with the 24-Period Data Set

```
function IAS09_THS24
emgen_THS24
loop  = 10;
PEM{1} = PEM0;
PEM{2} = PEM0;
PEM{3} = PEM0;
% Timeframe = 24
Pd = zeros(24,3);
Qd = zeros(24,3);
```

```
% Timeframe = 24
% Qref =  [0.089 0.0456 0.0559;
%         0.3329 0.3671 0.0897;
%         0.2231 0.6802 0.0499;
%         0.2326 0.6655 0.048;
%         0.3078 0.7959 0.0474];
%Pref = [11.2*ones(5,1) 10.5*ones(5,1) 13.2*ones(5,1)];

%===ref 0304 value ===
%Pref = [11.6 9.8 10.7 14.5 12.6];
%Qorg = [0.0989 0.0456 0.0799;0.2497 0.3059 0.0523;0.2479 0.6802
0.0712;0.2326 0.6655 0.048;0.2565 0.6632 0.0474];
%====================

%===ref THS 1 value===
%Pref =  [9.8 10.1279 9.8 9.8 9.8]'; %RT H5

%Qref = [0.554.*ones(1,3);0.4967.*ones(1,3); 0.531712.*ones(1,3);
%0.527694.*ones(1,3);0.557541.*ones(1,3)]; %RT H5
%Qorg = 0.5.*ones(5,3);

% Pref1 = 10.5.*ones(24,1);
Pref2 = 10.4.*ones(24,1);
% Pref3 = 10.3.*ones(24,1);
Pref1 = Pref2;
Pref3 = Pref2;
Pref = [Pref1 Pref2 Pref3];

% Qref1 = [0.7456    0.7059    0.6802    0.6655    0.6632    0.6885...
%         0.7386    0.8090    0.8748     0.9297    0.9611    0.9852...
%         0.9939    0.9961    0.9980    1.0000    0.9936    0.9644...
%         0.9033    0.8572    0.8400    0.8063    0.7600
0.7045]'/2;%nyc/2
Qref2 = [0.3255    0.3021    0.2890    0.2815    0.2793    0.2880
0.3027 ...
        0.3371    0.3758    0.4136    0.4431    0.4669    0.4855
0.4951 ...
        0.4992    0.5000    0.4981    0.4850    0.4559    0.4261
0.4222 ...
        0.3941    0.3523    0.3099]'; %long island
% Qref3 = [0.7269    0.6798    0.6481    0.6238    0.6165    0.6301...
%          0.6794    0.7530    0.8273    0.8895    0.9373    0.9460...
%          0.9656    0.9886    0.9917    1.0000    0.9908    0.9985...
%          0.9125    0.8682    0.8701    0.8160    0.7364
0.6652]'/2;%dunwod/2
Qref1 = Qref2;
Qref3 = Qref2;
Qref = [Qref1 Qref2 Qref3];
%====================

%Qmin = [0.9 1 0.7];
%Qmax = [1 1 1];

Qorg = Qref;
```

```
for kk = 1:loop
    if kk ==1
        Ch = Qorg;
        read_in(Ch);
        [ps,mcp] = call_ed;
%RT market
%         mcp = [mcp(1);9.8;10.1279;9.8;9.8;mcp(end)];
        [nhr,nsup] = size(ps);
    else
        read_in(Ch); %update the datafile
        [ps,mcp] = call_ed;
%RT market
%         mcp = [mcp(1);9.8;10.1279;9.8;9.8;mcp(end)];
    end
    ps_rec{kk}  = ps;
    ps = ps(2:nhr,:);
    mcp_rec{kk} = mcp;
    mcp = mcp(2:nhr,:);
    for i = 1:3
        Pd(:,i) = mcp - Pref(:,i);
        Qd(:,i) = PEM{i} * Pd(:,i);
        Q(:,i)  = Qd(:,i) + Qref(:,i);
 %        Q1(:,i) = Qorg(:,i).*Qmin(i);
 %        Q2(:,i) = Qorg(:,i).*Qmax(i);
    end
%     Q = max(Q, Q1);
%     Q = min(Q, Q2);
    Ch = Q;
    ch_rec{kk+1} = Q;
end
    ch_rec{1} = Qref;

for ll = 1:loop
    if ll ==1
    figure
    h1 = axes;
    set(gca,'XLim',[1,24],'YLim',[0,1])
    set(gca,'XTick',1:1:24,'YTick',0:0.1:1)
    set(get(gca,'XLabel'),'String','Hours')
    set(get(gca,'YLabel'),'String','Generation')
    hold on
    plot(ps_rec{1}(2:nhr,:))

    figure
    h2 = axes;
    set(gca,'XLim',[1,24],'YLim',[9.5,13])
    set(gca,'XTick',1:1:24,'YTick',9.5:0.5:13)
    set(get(gca,'XLabel'),'String','Hours')
    set(get(gca,'YLabel'),'String','Price')
    hold on
    plot(mcp_rec{1}(2:nhr,:))

    figure
    h3 = axes;
    set(gca,'XLim',[1,24],'YLim',[0.25,0.55])
    set(gca,'XTick',1:1:24,'YTick',0.25:0.02:0.55)
```

```
    set(get(gca,'XLabel'),'String','Hours')
    set(get(gca,'YLabel'),'String','Load')
    hold on
    plot(ch_rec{1}(1:nhr-1,:))
    else
%          plot(h1,ps_rec{ll}(2:nhr,:),'LineWidth',6)
         plot(h1,ps_rec{ll}(2:nhr,1),'-','LineWidth',6,'Color','blue')
         plot(h1,ps_rec{ll}(2:nhr,2),'--','LineWidth',6,'Color','green')
          plot(h1,ps_rec{ll}(2:nhr,3),'-.','LineWidth',6,'Color','red')
         plot(h2,mcp_rec{ll}(2:nhr,:),'LineWidth',6,'Color','green')
          plot(h3,ch_rec{ll}(1:nhr-1,1),'-
','LineWidth',6,'Color','magenta')
          plot(h3,ch_rec{ll}(1:nhr-1,2),'--
','LineWidth',6,'Color','yellow')
           plot(h3,ch_rec{ll}(1:nhr-1,3),'-
.','LineWidth',6,'Color','cyan')
    end
end
disp('end of loop')
saveas(h1,'J:\MS Thesis\Thesis\Chapter 4_figures\24_13_G_SHB.fig')
saveas(h2,'J:\MS Thesis\Thesis\Chapter 4_figures\24_13_p_SHB.fig')
saveas(h3,'J:\MS Thesis\Thesis\Chapter 4_figures\24_13_D_SHB.fig')

%% input interface
function read_in(Ch)

%write into a temporary file
fid00 = fopen('psatdata_24_THS_1.gms','r'); %initialize
fid11 = fopen('psatdata.gms','w+'); %create a temporary file
frewind(fid00);
while 1
    t00 = fgetl(fid00);
%    t11 = fgetl(fid11);
    if strcmp(t00,'$kill Ch'),
        break
    else
       fprintf(fid11,'%s\n',t00);
    end
end
%t11 = fgetl(fid11); %point to the next line of 'parameter Ch'
fprintf(fid11,'$kill %s\n','Ch');
fprintf(fid11,'parameter %s /\n','Ch');
[nH,nLd] = size(Ch);
for j = 1:nLd,
        fprintf(fid11,'%s%s.%d %f\n','H','0',j,Ch(1,j));
    for i = 1:nH,
        fprintf(fid11,'%s%d.%d %f\n','H',i,j,Ch(i,j));
%        t11 = fgetl(fid00);
    end
end
fprintf(fid11,'/;\n');
fprintf(fid11,'%s\n','$offempty');
%write back
fid00 = fopen('psatdata_24_THS_1.gms','w+');
fid11 = fopen('psatdata.gms','r'); %create a temporary file
frewind(fid11);
```

```
while 1
    t11 = fgetl(fid11);
    if strcmp(t11,'$offempty');%0429
        break
    else
        fprintf(fid00,'%s\n',t11);
    end
end
fprintf(fid00,'%s\n','$offempty');
fclose(fid11);
fclose(fid00);

%% ----------------------------------------------------------------
function varargout = call_ed
    status = 0;
    t0 = clock;
    [status,result] = system(['gams ','fm_THS_1.gms']);
    disp([' GAMS routine completed in ',num2str(etime(clock,t0)),' s'])
    if status
        disp(result)
        disp('Error!!')
    return
    end
    nout = 0;
    EPS = eps;
    clear psatsol
    psatsol
    if nout < nargout
        for i = nout+1:nargout
            varargout{i} = [];
        end
    end

    if nout > nargout
        varargout(nargout+1:nout) = [];
    end
```

Running the Proposed Bidding Mechanism in the HA Market with the 24-Period Data Set

```
% The real time operation of renewable energy
function IAS09_THS24_RT
% wind coefficient
% generated by random('wbl',1.*ones(24,1),7.*ones(24,1))
% generating PEM
emgen_THS24
PEM = PEM6;
%initial reference point
Pref = 10.4.*ones(24,1);
Pref = [Pref Pref Pref];
Qorg = [0.3255    0.3021    0.2890    0.2815    0.2793    0.2880
0.3027 ...
        0.3371    0.3758    0.4136    0.4431    0.4669    0.4855
0.4951 ...
        0.4992    0.5000    0.4981    0.4850    0.4559    0.4261
0.4222 ...
```

```
        0.3941    0.3523    0.3099]';
Qorg = [Qorg Qorg Qorg];
Qref = Qorg;

%simulating real-time market
for rt = 1:24
    display(rt);
    PEM_RT = zeros(24,24);
    PEM_RT(rt:end,rt:end) = PEM(rt:end,rt:end);
    RT_iter(PEM_RT,Pref,Qref,Qorg,WindCo(:,rt),rt)
end

%rt is the RT market index
%% compute market equilibrium in every hourly ahead market
function RT_iter(PEM,Pref,Qref,Qorg,WindCo,rt)

loop  = 10;

% Timeframe = 24
Pd = zeros(24,3);
Qd = zeros(24,3);
Gwind = [ones(24,1),WindCo,ones(24,1)];
read_in(Gwind,'Sup');

for kk = 1:loop
    if kk ==1
        Ch = Qorg;
        read_in(Ch,'Ch');
        [ps,mcp] = call_ed;
        [nhr,nsup] = size(ps);
    else
        read_in(Ch,'Ch'); %update the datafile
        [ps,mcp] = call_ed;
    end
    ps_rec{kk}  = ps;
    ps = ps(2:nhr,:);
    mcp_rec{kk} = mcp;
    mcp = mcp(2:nhr,:);
    for i = 1:3
        Pd(:,i) = mcp - Pref(:,i);
        Qd(:,i) = PEM * Pd(:,i);
        Q(:,i)  = Qd(:,i) + Qref(:,i);
 %       Q1(:,i) = Qorg(:,i).*Qmin(i);
 %       Q2(:,i) = Qorg(:,i).*Qmax(i);
    end
%    Q = max(Q, Q1);
%    Q = min(Q, Q2);
    Ch = Q;
    ch_rec{kk+1} = Q;
end
    ch_rec{1} = Qref;

for ll = 1:loop
    if ll ==1
    figure
    h1 = axes;
```

```
    set(gca,'XLim',[1,24],'YLim',[0,1])
    set(gca,'XTick',1:1:24,'YTick',0:0.1:1)
    set(get(gca,'XLabel'),'String','Hours')
    set(get(gca,'YLabel'),'String','Generation')
    hold on
    plot(ps_rec{1}(2:nhr,:))

    figure
    h2 = axes;
    set(gca,'XLim',[1,24],'YLim',[9.5,13])
    set(gca,'XTick',1:1:24,'YTick',9.5:0.5:13)
    set(get(gca,'XLabel'),'String','Hours')
    set(get(gca,'YLabel'),'String','Price')
    hold on
    plot(mcp_rec{1}(2:nhr,:))

    figure
    h3 = axes;
    set(gca,'XLim',[1,24],'YLim',[0.25,0.55])
    set(gca,'XTick',1:1:24,'YTick',0.25:0.02:0.55)
    set(get(gca,'XLabel'),'String','Hours')
    set(get(gca,'YLabel'),'String','Load')
    hold on
    plot(ch_rec{1})
    else
%        plot(h1,ps_rec{ll}(2:nhr,:),'LineWidth',6)
         plot(h1,ps_rec{ll}(2:nhr,1),'-','LineWidth',6,'Color','blue')
         plot(h1,ps_rec{ll}(2:nhr,2),'--','LineWidth',6,'Color','green')
         plot(h1,ps_rec{ll}(2:nhr,3),'-.','LineWidth',6,'Color','red')
         plot(h2,mcp_rec{ll}(2:nhr,:),'LineWidth',6,'Color','green')
         plot(h3,ch_rec{ll}(1:nhr-1,:),'LineWidth',6,'Color','yellow')
    end
end
disp('end of loop')

% ps = zeros(24,3);
% ch = zeros(24,1);
% mcp = zeros(24,1);
% fdcm =
0;                                                                     %in
itialize demand clearing market flag
% for nn = rt:24
%     dif = abs(mcp_rec{ll}(nn,:)- mcp_rec{ll-1}(nn,:));
%     if
~(dif)                                                                 %if
not convergent
% %   for renewable energy, G2 cost = 0. Thus unconvergency can only
caused demand clears the market
%
% %         if dif > max(10.7-9.8, 12.6-
10.7)                                  %unconvergency due to slope
value
% %             mcp(nn,:) =
10.7;                                              %the price is known
```

```
% %
syms(genvarname(['ch',num2str(nn)]))                           %claim
the load is unknown
% %
else                                                             %unc
onvergency due to demand clears the market
%                fdcm = 1;
%                ch(nn,:) = (1 +
WindCo(nn,rt)*0.7)/3;              %demand equals to the capacity
of (G1+G2)
%                mcp(nn,:)=
syms(genvarname(['p',num2str(nn)]));               %claim the price is
unknown
%     else
%        mcp(nn,:) =
mcp_rec{ll}(nn+1,:);                                    %the price as
the convergent one
%        ch(nn,:)=
syms(genvarname(['d',num2str(nn)]));                      %the demand
set as variable
%     end
% end
%
% if fdcm
%     f = (ch - Qref)- PEM*(mcp - Pref);
%     varargout =
solve(f(1),f(2),f(3),f(4),f(5),f(6),f(7),f(8),f(9),f(10),...
%
f(11),f(12),f(13),f(14),f(15),f(16),f(17),f(18),...
%                       f(19),f(20),f(21),f(22),f(23),f(24));
% end
figure
h4 = axes;
set(gca,'XLim',[1,24],'YLim',[0,1.5])
set(gca,'XTick',1:1:24,'YTick',0:0.1:1.5)
set(get(gca,'XLabel'),'String','Hours')
set(get(gca,'YLabel'),'String','Wind')
hold on
plot(h4,WindCo,'LineWidth',6,'Color','blue')

saveas(h1,['J:\MS Thesis\Thesis\Chapter
4_figures\24_14_G_raw',num2str(rt),'.fig'])
saveas(h2,['J:\MS Thesis\Thesis\Chapter
4_figures\24_14_p_raw',num2str(rt),'.fig'])
saveas(h3,['J:\MS Thesis\Thesis\Chapter
4_figures\24_14_D_raw',num2str(rt),'.fig'])
saveas(h4,['J:\MS Thesis\Thesis\Chapter
4_figures\24_14_wind',num2str(rt),'.fig'])
fclose('all') %close the files opened by "save"

%% input interface
function read_in(xx,xxname)

%write into a temporary file
fid00 = fopen('psatdata_24rt_THS_1.gms','r'); %initialize
fid11 = fopen('psatdata.gms','w+'); %create a temporary file
```

```
frewind(fid00);
while 1
    t00 = fgetl(fid00);
%     t11 = fgetl(fid11);
    if strcmp(t00,['$kill ',xxname]),
        break
    else
       fprintf(fid11,'%s\n',t00);
    end
end
%t11 = fgetl(fid11); %point to the next line of 'parameter Ch'
fprintf(fid11,'$kill %s\n',xxname);
fprintf(fid11,'parameter %s /\n',xxname);
t00 = fgetl(fid00);

[nH,nLd] = size(xx);
for j = 1:nLd,
        fprintf(fid11,'%s%s.%d %f\n','H','0',j,xx(1,j));
        t00 = fgetl(fid00);
    for i = 1:nH,
        fprintf(fid11,'%s%d.%d %f\n','H',i,j,xx(i,j));
%         t11 = fgetl(fid00);
        t00 = fgetl(fid00);
    end
end
fprintf(fid11,'/;\n');
t00 = fgetl(fid00);

while 1
    t00 = fgetl(fid00);
     if strcmp(t00,'$offempty'),
        break
    else
       fprintf(fid11,'%s\n',t00);
    end
end
fprintf(fid11,'%s\n','$offempty');

%write back
fid00 = fopen('psatdata_24rt_THS_1.gms','w+');
fid11 = fopen('psatdata.gms','r'); %create a temporary file
frewind(fid11);
while 1
    t11 = fgetl(fid11);
    if strcmp(t11,'$offempty');%0429
        break
    else
        fprintf(fid00,'%s\n',t11);
    end
end
fprintf(fid00,'%s\n','$offempty');
fclose(fid11);
fclose(fid00);

%% ------------------------------------------------------------------
function varargout = call_ed
```

```
    status = 0;
    t0 = clock;
    [status,result] = system(['gams ','fm_THS_1.gms']);
    disp([' GAMS routine completed in ',num2str(etime(clock,t0)),' s'])
    if status
        disp(result)
        disp('Error!!')
    return
    end
    nout = 0;
    EPS = eps;
    clear psatsol
    psatsol
    if nout < nargout
        for i = nout+1:nargout
            varargout{i} = [];
        end
    end

    if nout > nargout
        varargout(nargout+1:nout) = [];
    end
```

The Optimization Problem Formulation of the Proposed Bidding Mechanism

```
$title GAMS/PSAT interface for solving the electricty market problem
*
=======================================================================
=======
$onempty
$offlisting
$offupper

*
=======================================================================
=======
* include data created with the PSAT-GAMS Interface
$if exist psatglobs_24_THS_1.gms $include psatglobs_24_THS_1.gms
*
=======================================================================
=======

sets B     index of buses        /1*%nBus%/,
     L     index of lines        /1*%nLine%/,
     G     index of suppliers    /1*%nPs%/,
     C     index of consumers    /1*%nPd%/,
     SW    index of slack buses  /1*%nSW%/,
     PV    index of pv buses     /1*%nPV%/,
     H     index of hours        /H0*%nH%/,
     Day   index of days         /D1*D7/,
     Week  index of weeks        /W1*W52/,
     Br(B) reference bus index   /%nBusref%/;

sets supply /Ps0, Psmax, Psmin, Cs, suc,
            mut, mdt, rut, rdt, u0, y0, z0/,
     demand /Pd0, Pdmax, Pdmin, tgphi, Cd/,
```

```
     data    /V0, t0, Pg0, Qg0, Pl0, Ql0,
              Qgmax, Qgmin, Vmax, Vmin, ksw, kpv/,
     lines   /g, b, g0, b0, Pijmax, Pjimax/,
     days    /wwkdy, wwknd, swkdy, swknd, sfwkdy, sfwknd/,
     vsc     /lmin, lmax, omega, line/;

alias (B,BB);
alias (H,HH);
alias (G,GG);

parameters S(G,supply)     supply data //,
           D(C,demand)     demand data //,
           X(B,data)       network data //,
           N(L,lines)      line data //,
           lambda(vsc)     loading paramter //,
           Ps_idx(B,G)     supply incidence matrix //,
           Pd_idx(B,C)     demand incidence matrix //,
           SW_idx(B,SW)    slack bus incidence matrix //,
           PV_idx(B,PV)    PV bus incidence matrix //,
           Ch(H,C)         charge profile //,
           Sup(H,G)        supply profile //,
           CC(H,G)         generation cost changing coefficient //,
           Li(L,B)         node-to-branch (ij) incidence matrix //,
           Lj(L,B)         node-to-branch (ji) incidence matrix //,
           Gh(B,BB)        conductance matrix //,
           Bh(B,BB)        admittance matrix //,
           Ghc(B,BB)       conductance matrix (critical system) //,
           Bhc(B,BB)       admittance matrix (critical system) //;

scalars MLC   maximum loading condition /0/,
        pi    /3.1416/
        T     /0/;

*
======================================================================
=======
* include data created with the PSAT-GAMS Interface
$if exist psatdata_24rt_THS_1.gms $include psatdata_24rt_THS_1.gms
*
======================================================================
=======

T = card(H)-1;

*
======================================================================
=======
* ==================== C O M M O N   V A R I A B L E S
=======================
*
======================================================================
=======

variables obj value to be minimized,
          Pij(H,L) flows from bus i to bus j,
```

```
          Pji(H,L) flows from bus j to bus i,
          V(H,B) bus voltage magnitudes,
          a(H,B) bus voltage phases,
          Ps(H,G) power supply bids,
          Pd(H,C) power demand bids,
          Qg(H,B) generator reactive powers,
          u(H,G)  1 if gen. G is committed in hour H,
          y(H,G)  1 if gen. G is started-up at the beginning of hour H,
          z(H,G)  1 if gen. G is shut-down at the beginning of hour H;

positive variables z(H,G);

binary variables u(H,G),
                 y(H,G);

* ----------------------------------------------------------------------
---------
* initial values
* ----------------------------------------------------------------------
---------


Qg.l(H,B) = X(B,'Qg0');
V.l(H,B) = X(B,'V0');
a.l(H,B) = X(B,'t0');
Ps.l(H,G) = Sup(H,G)*0.5*(S(G,'Psmax')+S(G,'Psmin'));
Pd.l(H,C) = Ch(H,C)*0.5*(D(C,'Pdmax')+D(C,'Pdmin'));

* ----------------------------------------------------------------------
---------
* zero-one variable initialization
* ----------------------------------------------------------------------
---------


z.up(H,G) = 1;
u.up(H,G) = 1;
y.up(H,G) = 1;
z.lo(H,G) = 0;
u.lo(H,G) = 0;
y.lo(H,G) = 0;
S(G,'u0') = 0$(S(G,'y0')<=0)+1$(S(G,'y0')>=1);
S(G,'z0') = S(G,'mdt')+1;

* ----------------------------------------------------------------------
---------
* limits
* ----------------------------------------------------------------------
---------


* Bid Blocks
Pd.up(H,C) = Ch(H,C)*D(C,'Pdmax');
Pd.lo(H,C) = Ch(H,C)*D(C,'Pdmin');

* Voltages & Voltage Limits
V.up(H,B) = X(B,'Vmax');
V.lo(H,B) = X(B,'Vmin');
```

```
a.up(H,B) = pi;
a.lo(H,B) = -pi;

* Generator Reactive Power Limits
Qg.up(H,B) = X(B,'Qgmax');
Qg.lo(H,B) = X(B,'Qgmin');

* Flow limits on transmission lines
Pij.up(H,L) =  N(L,'Pijmax');
Pij.lo(H,L) = -N(L,'Pijmax');
Pji.up(H,L) =  N(L,'Pijmax');
Pji.lo(H,L) = -N(L,'Pijmax');

* ----------------------------------------------------------------------
---------
* define equations
* ----------------------------------------------------------------------
---------

equation cost             objective function,
         pmaxlim(H,G)     maximum power supply output,
         pminlim(H,G)     minimum power supply output,
         logicupdn1(H,G)  start-up and shut-down and running logic 1,
         logicupdn2(H,G)  start-up and shut-down and running logic 2,
         rampdown(H,G)    maximum ramp down rate limit,
         rampup(H,G)      maximum ramp up rate limit,
         uptime1(G)       mimimum up time logic 1,
         uptime2(H,G)     mimimum up time logic 2,
         uptime3(H,G)     mimimum up time logic 3,
         dwntime1(G)      mimimum down time logic 1,
         dwntime2(H,G)    mimimum down time logic 2,
         dwntime3(H,G)    mimimum down time logic 3;

* ----------------------------------------------------------------------
---------
* maximum and minimum power supply output constraints
* ----------------------------------------------------------------------
---------
pmaxlim(H,G)$(S(G,'Psmax') and (ord(H) gt 1)).. Ps(H,G) =l=
Sup(H,G)*S(G,'Psmax');
pminlim(H,G)$(S(G,'Psmin') and (ord(H) gt 1)).. Ps(H,G) =g=
Sup(H,G)*S(G,'Psmin');

* ----------------------------------------------------------------------
---------
* logic up and logic down
* ----------------------------------------------------------------------
---------
*logicupdn1(H,G)$(ord(H) gt 1).. y(H,G)-z(H,G) =e= u(H,G)-u(H-1,G);
*logicupdn2(H,G)$(ord(H) gt 1).. y(H,G)+z(H,G) =l= 1;

* ----------------------------------------------------------------------
---------
* ramp up and ramp down
```

```
* ----------------------------------------------------------------------
---------
*rampdown(H,G)$(S(G,'rdt') and (ord(H) gt 1)).. Ps(H-1,G)-Ps(H,G) =l=
*                                             S(G,'rdt');


*rampup(H,G)$(S(G,'rut') and (ord(H) gt 1)).. Ps(H,G)-Ps(H-1,G) =l=
*                                             S(G,'rut');


* ----------------------------------------------------------------------
---------
* up time constraints
* ----------------------------------------------------------------------
---------
*uptime1(G).. sum(H$((ord(H) gt 1) and (ord(H) le
*              min(T,(S(G,'mut')-S(G,'y0'))*S(G,'u0'))+1)),1-u(H,G))=e=0;

*uptime2(H,G)$((ord(H) gt (min(T,(S(G,'mut')-S(G,'y0'))*S(G,'u0'))+1))
and
*        (ord(H) le T-S(G,'mut')+1+1))..
*        sum(HH$((ord(HH) ge ord(H)) and (ord(HH) le
*        ord(H)+S(G,'mut')-1)),u(HH,G))=g=S(G,'mut')*y(H,G);

*uptime3(H,G)$((ord(H) gt T-S(G,'mut')+2) and (ord(H) le T+1))..
*        sum(HH$((ord(HH) ge ord(H)) and (ord(HH) le T+1)),u(HH,G)-
y(H,G))=g=0;

* ----------------------------------------------------------------------
---------
* down time constraints
* ----------------------------------------------------------------------
---------
*dwntime1(G).. sum(H$((ord(H) gt 1) and (ord(H) le
*              min(T,(S(G,'mdt')-S(G,'z0'))*(1-
S(G,'u0')))+1)),u(H,G))=e=0;

*dwntime2(H,G)$((ord(H) gt min(T,(S(G,'mdt')-S(G,'z0'))*(1-
S(G,'u0')))+1)
*        and (ord(H) le T-S(G,'mdt')+1+1))..
*        sum(HH$((ord(HH) ge ord(H)) and (ord(HH) le
*        ord(H)+S(G,'mdt')-1)),1-u(HH,G))=g=S(G,'mdt')*z(H,G);

*dwntime3(H,G)$((ord(H) gt T-S(G,'mdt')+2) and (ord(H) le T+1))..
*        sum(HH$((ord(HH) ge ord(H)) and (ord(HH) le T+1)),1-u(HH,G)-
z(H,G))=g=0;

* ----------------------------------------------------------------------
---------

$if %control% == 1 $goto jfloweq

equation Peq(H,B),
         Thetaref(H,B);
```

```
$if %control% == 2 $goto jfloweq

equations Qeq(H,B);

$if %flow% == 0 $goto jfloweq

equations Pijeq(H,L),
          Pjieq(H,L);

$label jfloweq

*
==========================================================================
=======
* -----------------------------------------------------------------------
---------
* ============================ M E T H O D S
===================================
* -----------------------------------------------------------------------
---------
*
==========================================================================
=======

* check method
$if %control% == 1 $goto auction
$if %control% == 2 $goto mcm
$if %control% == 3 $goto opf
$if %control% == 4 $goto vscopf
$if %control% == 5 $goto mlcopf

*
==========================================================================
=======
* ====================== S I M P L E   A U C T I O N
========================
*
==========================================================================
=======

$label auction

* -----------------------------------------------------------------------
---------
* objective function
cost.. obj =e= sum(H$(ord(H) gt 1),
       sum(G,CC(H,G)*Ps(H,G)*S(G,'Cs')));
* -----------------------------------------------------------------------
---------

equation Pbalance(H);
Pbalance(H).. sum(G,Ps(H,G)) - sum(C,Ch(H,C)) =e= 0;
```

```
$goto solvestat

*
==========================================================================
=======
* =========== M A R K E T   C L E A R I N G   M E C H A N I S M
================
*
==========================================================================
=======

$label mcm

* -----------------------------------------------------------------------
---------
* objective function
cost.. obj =e= sum(H$(ord(H) gt 1),
       sum(G,Ps(H,G)*S(G,'Cs'))) +
       sum(H$(ord(H) gt 1),sum(G,y(H,G)*S(G,'suc')));
* -----------------------------------------------------------------------
---------

Peq(H,B).. sum(G,Ps_idx(B,G)*Ps(H,G)) - sum(C,Pd_idx(B,C)*Ch(H,C)) -
           sum(BB,Bh(B,BB)*a(H,BB)) =e= 0;

equation Pijeq(H,L);
Pijeq(H,L).. Pij(H,L) =e= sum(B,Li(L,B)*a(H,B))-sum(B,Lj(L,B)*a(H,B));

Thetaref(H,B)$Br(B).. a(H,B) =e= 0;

$goto solvestat

*
==========================================================================
=======
* ================== O P T I M A L   P O W E R   F L O W
======================
*
==========================================================================
=======

$label opf

* -----------------------------------------------------------------------
---------
* objective function
cost.. obj =e= sum(H,sum(G,Ps(H,G)*S(G,'Cs')) -
sum(C,Pd(H,C)*D(C,'Cd'))) +
       sum(H$(ord(H) gt 1),sum(G,y(H,G)*S(G,'suc')));
* -----------------------------------------------------------------------
---------

Peq(H,B).. sum(G,Ps_idx(B,G)*Ps(H,G)) - sum(C,Pd_idx(B,C)*Pd(H,C)) +
           Ch(H)*X(B,'Pg0') - Ch(H)*X(B,'Pl0') -
           V(H,B)*sum(BB,V(H,BB)*(Gh(B,BB)*cos(a(H,B)-a(H,BB)) +
```

```
          Bh(B,BB)*sin(a(H,B)-a(H,BB)))) =e= 0;

Qeq(H,B).. Qg(H,B) - Ch(H)*X(B,'Ql0') -
          sum(C,Pd_idx(B,C)*(D(C,'tgphi')*Pd(H,C)))
          - V(H,B)*sum(BB,V(H,BB)*(Gh(B,BB)*sin(a(H,B)-a(H,BB)) -
          Bh(B,BB)*cos(a(H,B)-a(H,BB)))) =e= 0;

Thetaref(H,B)$Br(B).. a(H,B) =e= 0;

*MLCeq.. MLC0 - (1+lambda)*(sum(C,Pd(C))+sum(B,X(B,'Pl0'))) =g= 0;

$goto floweq

*
=========================================================================
=======
* ====== V O L T A G E   S T A B I L I T Y   C O N S T R A I N E D
===========
* ================== O P T I M A L   P O W E R   F L O W
======================
*
=========================================================================
=======

$label vscopf

* -----------------------------------------------------------------------
---------
* local variables
* -----------------------------------------------------------------------
---------

variables Pijc(H,L),
          Pjic(H,L),
          Vc(H,B),
          ac(H,B),
          Qgc(H,B),
          kg(H),
          lambdac(H);

* -----------------------------------------------------------------------
---------
* initial values
* -----------------------------------------------------------------------
---------

Qgc.l(H,B) = X(B,'Qg0');
Vc.l(H,B)  = X(B,'V0');
ac.l(H,B)  = X(B,'t0');
kg.l(H) = 0;

* -----------------------------------------------------------------------
---------
* limits
```

```
* ---------------------------------------------------------------------------
---------

* Voltages & Voltage Limits
Vc.up(H,B) = X(B,'Vmax');
Vc.lo(H,B) = X(B,'Vmin');
ac.up(H,B) =  pi;
ac.lo(H,B) = -pi;
kg.lo(H) = -1;
kg.up(H) =  1;

* Generator Reactive Power Limits
Qgc.up(H,B) = X(B,'Qgmax');
Qgc.lo(H,B) = X(B,'Qgmin');

* Flow limits on transmission lines
Pijc.up(H,L) =  N(L,'Pijmax');
Pijc.lo(H,L) = -N(L,'Pijmax');
Pjic.up(H,L) =  N(L,'Pijmax');
Pjic.lo(H,L) = -N(L,'Pijmax');

lambdac.up(H) = lambda('lmax');
lambdac.lo(H) = lambda('lmin');

* ---------------------------------------------------------------------------
---------
* objective function
cost.. obj =e= (1-lambda('omega'))*sum(H,sum(G,Ps(H,G)*S(G,'Cs')) -
       sum(C,Pd(H,C)*D(C,'Cd'))) +
       (1-lambda('omega'))*sum(H$(ord(H)  gt  1),sum(G,y(H,G)*S(G,'suc')))
-
       sum(H,lambda('omega')*lambdac(H));
* ---------------------------------------------------------------------------
---------

equations Pceq(H,B),
          Qceq(H,B),
          Thetacref(H,B);

$if %flow% == 0 $goto jfloweqc

equations Pijceq(H,L),
          Pjiceq(H,L);

$label jfloweqc

Peq(H,B)..  sum(G,Ps_idx(B,G)*Ps(H,G)) -
            sum(C,Pd_idx(B,C)*Pd(H,C)) +
            Ch(H)*X(B,'Pg0') - Ch(H)*X(B,'Pl0') -
            V(H,B)*sum(BB,V(H,BB)*(Gh(B,BB)*cos(a(H,B)-a(H,BB)) +
            Bh(B,BB)*sin(a(H,B)-a(H,BB)))) =e= 0;

Qeq(H,B)..  Qg(H,B) - Ch(H)*X(B,'Ql0') -
            sum(C,Pd_idx(B,C)*(D(C,'tgphi')*Pd(H,C)))
            - V(H,B)*sum(BB,V(H,BB)*(Gh(B,BB)*sin(a(H,B)-a(H,BB)) -
```

```
            Bh(B,BB)*cos(a(H,B)-a(H,BB)))) =e= 0;

Thetaref(H,B)$Br(B).. a(H,B) =e= 0;

Pceq(H,B).. (1+lambdac(H)+kg(H))*(sum(G,Ps_idx(B,G)*Ps(H,G)) +
            Ch(H)*X(B,'Pg0'))
            - (1+lambdac(H))*(sum(C,Pd_idx(B,C)*Pd(H,C)) +
            Ch(H)*X(B,'Pl0'))
            - Vc(H,B)*sum(BB,Vc(H,BB)*(Ghc(B,BB)*cos(ac(H,B)-ac(H,BB))
+
            Bhc(B,BB)*sin(ac(H,B)-ac(H,BB)))) =e= 0;

Qceq(H,B).. Qgc(H,B) - (1+lambdac(H))*(Ch(H)*X(B,'Ql0') +
            sum(C,Pd_idx(B,C)*(D(C,'tgphi')*Pd(H,C)))) -
            Vc(H,B)*sum(BB,Vc(H,BB)*(Ghc(B,BB)*sin(ac(H,B)-ac(H,BB)) -
            Bhc(B,BB)*cos(ac(H,B)-ac(H,BB)))) =e= 0;

Thetacref(H,B)$Br(B).. ac(H,B) =e= 0;

$goto floweq

*
======================================================================
=======
* =========== M A X I M U M   L O A D I N G   C O N D I T I O N
================
*
======================================================================
=======

$label mlcopf

* ---------------------------------------------------------------------
---------
* local variables
* ---------------------------------------------------------------------
---------

variables kg(H),
          lambdac(H);

* ---------------------------------------------------------------------
---------
* initial values
* ---------------------------------------------------------------------
---------

kg.l(H) = 0;
lambdac.l(H) = 1;

* ---------------------------------------------------------------------
---------
* limits
```

```
* ----------------------------------------------------------------------
---------

* Bid Blocks
Ps.up(H,G) = S(G,'Ps0');
Pd.up(H,C) = Ch(H)*D(C,'Pd0');
Ps.lo(H,G) = S(G,'Ps0');
Pd.lo(H,C) = Ch(H)*D(C,'Pd0');
* Loading Parameter
kg.lo(H) = -1;
kg.up(H) =  1;
lambdac.lo(H) = 0;

* ----------------------------------------------------------------------
---------
* objective function
cost.. obj =e= sum(H,-lambdac(H));
* ----------------------------------------------------------------------
---------

Peq(H,B).. (lambdac(H)+kg(H))*(sum(G,Ps_idx(B,G)*Ps(H,G)) +
           Ch(H)*X(B,'Pg0')) -
           lambdac(H)*(sum(C,Pd_idx(B,C)*Pd(H,C)) + Ch(H)*X(B,'Pl0')) -
           V(H,B)*sum(BB,V(H,BB)*(Gh(B,BB)*cos(a(H,B)-a(H,BB)) +
           Bh(B,BB)*sin(a(H,B)-a(H,BB)))) =e= 0;

Qeq(H,B).. Qg(H,B) - lambdac(H)*(Ch(H)*X(B,'Ql0') +
           sum(C,Pd_idx(B,C)*(D(C,'tgphi')*Pd(H,C)))) -
           V(H,B)*sum(BB,V(H,BB)*(Gh(B,BB)*sin(a(H,B)-a(H,BB)) -
           Bh(B,BB)*cos(a(H,B)-a(H,BB)))) =e= 0;

Thetaref(H,B)$Br(B).. a(H,B) =e= 0;

* ----------------------------------------------------------------------
---------
* flow limit equations
* ----------------------------------------------------------------------
---------

$label floweq

$if %flow% == 0 $goto solvestat
$if %flow% == 1 $goto iflows
$if %flow% == 2 $goto pflows
$if %flow% == 3 $goto sflows

$label pflows

Pijeq(H,L).. Pij(H,L) =e= N(L,'g0')*sum(B,Li(L,B)*V(H,B)*V(H,B)) -
             N(L,'b')*sum(B,Li(L,B)*V(H,B)*sin(a(H,B)))*
             sum(B,Lj(L,B)*V(H,B)*cos(a(H,B))) +
             N(L,'b')*sum(B,Li(L,B)*V(H,B)*cos(a(H,B)))*
             sum(B,Lj(L,B)*V(H,B)*sin(a(H,B))) -
             N(L,'g')*sum(B,Li(L,B)*V(H,B)*cos(a(H,B)))*
             sum(B,Lj(L,B)*V(H,B)*cos(a(H,B))) -
```

```
            N(L,'g')*sum(B,Li(L,B)*V(H,B)*sin(a(H,B)))*
            sum(B,Lj(L,B)*V(H,B)*sin(a(H,B)));

Pjieq(H,L).. Pji(H,L) =e= N(L,'g0')*sum(B,Lj(L,B)*V(H,B)*V(H,B)) -
            N(L,'b')*sum(B,Lj(L,B)*V(H,B)*sin(a(H,B)))*
            sum(B,Li(L,B)*V(H,B)*cos(a(H,B))) +
            N(L,'b')*sum(B,Lj(L,B)*V(H,B)*cos(a(H,B)))*
            sum(B,Li(L,B)*V(H,B)*sin(a(H,B))) -
            N(L,'g')*sum(B,Lj(L,B)*V(H,B)*cos(a(H,B)))*
            sum(B,Li(L,B)*V(H,B)*cos(a(H,B))) -
            N(L,'g')*sum(B,Lj(L,B)*V(H,B)*sin(a(H,B)))*
            sum(B,Li(L,B)*V(H,B)*sin(a(H,B)));

$if %control% == 3 $goto solvestat
$if %control% == 5 $goto solvestat

N(L,'b')$(ord(L) eq lambda('line')) = -1E-6;
N(L,'g')$(ord(L) eq lambda('line')) = 0;
N(L,'b0')$(ord(L) eq lambda('line')) = 0;
N(L,'g0')$(ord(L) eq lambda('line')) = 0;

Pijceq(H,L).. Pijc(H,L) =e= N(L,'g0')*sum(B,Li(L,B)*Vc(H,B)*Vc(H,B)) -
            N(L,'b')*sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B)))*
            sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B))) +
            N(L,'b')*sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B)))*
            sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B))) -
            N(L,'g')*sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B)))*
            sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B))) -
            N(L,'g')*sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B)))*
            sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B)));

Pjiceq(H,L).. Pjic(H,L) =e= N(L,'g0')*sum(B,Lj(L,B)*Vc(H,B)*Vc(H,B)) -
            N(L,'b')*sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B)))*
            sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B))) +
            N(L,'b')*sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B)))*
            sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B))) -
            N(L,'g')*sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B)))*
            sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B))) -
            N(L,'g')*sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B)))*
            sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B)));

$goto solvestat

$label iflows

Pijeq(H,L).. Pij(H,L) =e= sqrt(
            sqr(N(L,'g')*sum(B,Li(L,B)*V(H,B)*cos(a(H,B)))-
            N(L,'g')*sum(B,Lj(L,B)*V(H,B)*cos(a(H,B)))-
            N(L,'b')*sum(B,Li(L,B)*V(H,B)*sin(a(H,B)))+
            N(L,'b')*sum(B,Lj(L,B)*V(H,B)*sin(a(H,B)))-
            N(L,'b0')*sum(B,Li(L,B)*V(H,B)*sin(a(H,B))))+
            sqr(N(L,'b')*sum(B,Li(L,B)*V(H,B)*cos(a(H,B)))-
            N(L,'b')*sum(B,Lj(L,B)*V(H,B)*cos(a(H,B)))+
            N(L,'g')*sum(B,Li(L,B)*V(H,B)*sin(a(H,B)))-
            N(L,'g')*sum(B,Lj(L,B)*V(H,B)*sin(a(H,B)))+
```

```
              N(L,'b0')*sum(B,Li(L,B)*V(H,B)*cos(a(H,B)))));

Pjieq(H,L).. Pji(H,L) =e= sqrt(
              sqr(N(L,'g')*sum(B,Lj(L,B)*V(H,B)*cos(a(H,B)))-
              N(L,'g')*sum(B,Li(L,B)*V(H,B)*cos(a(H,B)))-
              N(L,'b')*sum(B,Lj(L,B)*V(H,B)*sin(a(H,B)))+
              N(L,'b')*sum(B,Li(L,B)*V(H,B)*sin(a(H,B)))-
              N(L,'b0')*sum(B,Lj(L,B)*V(H,B)*sin(a(H,B))))+
              sqr(N(L,'b')*sum(B,Lj(L,B)*V(H,B)*cos(a(H,B)))-
              N(L,'b')*sum(B,Li(L,B)*V(H,B)*cos(a(H,B)))+
              N(L,'g')*sum(B,Lj(L,B)*V(H,B)*sin(a(H,B)))-
              N(L,'g')*sum(B,Li(L,B)*V(H,B)*sin(a(H,B)))+
              N(L,'b0')*sum(B,Lj(L,B)*V(H,B)*cos(a(H,B)))));

$if %control% == 3 $goto solvestat
$if %control% == 5 $goto solvestat

N(L,'b')$(ord(L) eq lambda('line')) = -1E-6;
N(L,'g')$(ord(L) eq lambda('line')) = 0;
N(L,'b0')$(ord(L) eq lambda('line')) = 0;
N(L,'g0')$(ord(L) eq lambda('line')) = 0;

Pijceq(H,L).. Pijc(H,L) =e= sqrt(
               sqr(N(L,'g')*sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B)))-
               N(L,'g')*sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B)))-
               N(L,'b')*sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B)))+
               N(L,'b')*sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B)))-
               N(L,'b0')*sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B))))+
               sqr(N(L,'b')*sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B)))-
               N(L,'b')*sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B)))+
               N(L,'g')*sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B)))-
               N(L,'g')*sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B)))+
               N(L,'b0')*sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B)))));

Pjiceq(H,L).. Pjic(H,L) =e= sqrt(
               sqr(N(L,'g')*sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B)))-
               N(L,'g')*sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B)))-
               N(L,'b')*sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B)))+
               N(L,'b')*sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B)))-
               N(L,'b0')*sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B))))+
               sqr(N(L,'b')*sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B)))-
               N(L,'b')*sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B)))+
               N(L,'g')*sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B)))-
               N(L,'g')*sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B)))+
               N(L,'b0')*sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B)))));

$goto solvestat

$label sflows

Pijeq(H,L).. Pij(H,L) =e= sum(B,Li(L,B)*V(H,B))*sqrt(
              sqr(N(L,'g')*sum(B,Li(L,B)*V(H,B)*cos(a(H,B)))-
              N(L,'g')*sum(B,Lj(L,B)*V(H,B)*cos(a(H,B)))-
              N(L,'b')*sum(B,Li(L,B)*V(H,B)*sin(a(H,B)))+
              N(L,'b')*sum(B,Lj(L,B)*V(H,B)*sin(a(H,B)))-
```

```
             N(L,'b0')*sum(B,Li(L,B)*V(H,B)*sin(a(H,B))))+
             sqr(N(L,'b')*sum(B,Li(L,B)*V(H,B)*cos(a(H,B)))-
             N(L,'b')*sum(B,Lj(L,B)*V(H,B)*cos(a(H,B)))+
             N(L,'g')*sum(B,Li(L,B)*V(H,B)*sin(a(H,B)))-
             N(L,'g')*sum(B,Lj(L,B)*V(H,B)*sin(a(H,B)))+
             N(L,'b0')*sum(B,Li(L,B)*V(H,B)*cos(a(H,B)))));

Pjieq(H,L).. Pji(H,L) =e= sum(B,Lj(L,B)*V(H,B))*sqrt(
             sqr(N(L,'g')*sum(B,Lj(L,B)*V(H,B)*cos(a(H,B)))-
             N(L,'g')*sum(B,Li(L,B)*V(H,B)*cos(a(H,B)))-
             N(L,'b')*sum(B,Lj(L,B)*V(H,B)*sin(a(H,B)))+
             N(L,'b')*sum(B,Li(L,B)*V(H,B)*sin(a(H,B)))-
             N(L,'b0')*sum(B,Lj(L,B)*V(H,B)*sin(a(H,B))))+
             sqr(N(L,'b')*sum(B,Lj(L,B)*V(H,B)*cos(a(H,B)))-
             N(L,'b')*sum(B,Li(L,B)*V(H,B)*cos(a(H,B)))+
             N(L,'g')*sum(B,Lj(L,B)*V(H,B)*sin(a(H,B)))-
             N(L,'g')*sum(B,Li(L,B)*V(H,B)*sin(a(H,B)))+
             N(L,'b0')*sum(B,Lj(L,B)*V(H,B)*cos(a(H,B)))));

$if %control% == 3 $goto solvestat
$if %control% == 5 $goto solvestat

N(L,'b')$(ord(L) eq lambda('line')) = -1E-6;
N(L,'g')$(ord(L) eq lambda('line')) = 0;
N(L,'b0')$(ord(L) eq lambda('line')) = 0;
N(L,'g0')$(ord(L) eq lambda('line')) = 0;

Pijceq(H,L).. Pijc(H,L) =e= sum(B,Li(L,B)*Vc(H,B))*sqrt(
              sqr(N(L,'g')*sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B)))-
              N(L,'g')*sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B)))-
              N(L,'b')*sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B)))+
              N(L,'b')*sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B)))-
              N(L,'b0')*sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B))))+
              sqr(N(L,'b')*sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B)))-
              N(L,'b')*sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B)))+
              N(L,'g')*sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B)))-
              N(L,'g')*sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B)))+
              N(L,'b0')*sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B)))));

Pjiceq(H,L).. Pjic(H,L) =e= sum(B,Lj(L,B)*Vc(H,B))*sqrt(
              sqr(N(L,'g')*sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B)))-
              N(L,'g')*sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B)))-
              N(L,'b')*sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B)))+
              N(L,'b')*sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B)))-
              N(L,'b0')*sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B))))+
              sqr(N(L,'b')*sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B)))-
              N(L,'b')*sum(B,Li(L,B)*Vc(H,B)*cos(ac(H,B)))+
              N(L,'g')*sum(B,Lj(L,B)*Vc(H,B)*sin(ac(H,B)))-
              N(L,'g')*sum(B,Li(L,B)*Vc(H,B)*sin(ac(H,B)))+
              N(L,'b0')*sum(B,Lj(L,B)*Vc(H,B)*cos(ac(H,B)))));

$goto solvestat

* --------------------------------------------------------------------
---------
```

```
* Solve Market Problem
* ----------------------------------------------------------------------
---------

$label solvestat
model market /cost,pmaxlim,pminlim,pbalance/;

option iterlim = 100000

$if %control% == 1 $goto linearmodel
$if %control% == 2 $goto linearmodel
$if %control% == 3 $goto nonlinearmodel
$if %control% == 4 $goto nonlinearmodel
$if %control% == 5 $goto nonlinearmodel
$if %control% == 6 $goto nonlinearmodel

$label linearmodel

solve market using lp minimizing obj;
$goto skipone
parameters upar(H,G);

upar(H,G) = u.l(H,G);

equations cost2;

cost2.. obj =e= sum(H,sum(G,Ps(H,G)*S(G,'Cs')) -
sum(C,Pd(H,C)*D(C,'Cd')));

Ps.up(H,G) = S(G,'Psmax')*upar(H,G);
Ps.lo(H,G) = S(G,'Psmin')*upar(H,G);

$if %control% == 1
model market2 /cost2,Pbalance/;

$if %control% == 2
model market2 /cost2,Peq,Pijeq,Thetaref/;

solve market2 using lp minimizing obj;
$label skipone
$goto psatoutput

$label nonlinearmodel

solve market using minlp minimizing obj;

$if not %control% == 4 $goto psatoutput

lambdac.up = lambdac.l;
lambdac.lo = lambdac.l-1.0E-5;
lambda('omega') = 0;

solve market using minlp minimizing obj;
```

```
$label psatoutput

$libinclude psatout Ps.l H G

$if %control% == 1 $libinclude psatout Pbalance.m H
$if %control% == 1 $goto end_output

$if %control% == 2 $libinclude psatout Peq.m H B
$if %control% == 2 $goto end_output

$libinclude psatout V.l H B
$libinclude psatout a.l H B
$libinclude psatout Qg.l H B
$libinclude psatout Peq.m H B
$libinclude psatout Pij.l H L
$libinclude psatout Pji.l H L

$if %control% == 5 $goto no_dual

$libinclude psatout V.m H B
$libinclude psatout Pij.m H L
$libinclude psatout Pji.m H L

$label no_dual

$if %control% == 3 $goto end_output

$libinclude psatout lambdac.l H
$libinclude psatout kg.l H

$if %control% == 5 $goto end_output

$libinclude psatout Vc.l H B
$libinclude psatout ac.l H B
$libinclude psatout Qgc.l H B
$libinclude psatout Pijc.l H L
$libinclude psatout Pjic.l H L

$label end_output
```

Bibliography

1. Cardell, J.B., *integrating small scale distributed generation into a deregulated market: control strategies and price feedback*, in *electrical engineering and computer science*. 1997, MIT. p. 232.
2. Chacra, F.A., et al., *impact of energy storage costs on economical performance in a distribution substation.* IEEE transaction on power systems, 2005. **20**(2): p. 8.
3. DOE, *Benefits of demand response in electricity markets and recommendations for achieving them: a report to the United States Congress Pursuant to section 1252 of the Energy Policy Act of 2005.* 2006, U.S. Department of Energy.
4. David, A.K., *Dispatch methodologies for open access transmission systems.* IEEE transaction on power systems, 1998. **13**(1).
5. Zhang, L., et al., *Day-ahead generation scheduling with demand response*, in *IEEE/PES Transmission and distribution conference & exhibition: Asia and Pacific*. 2005: Dalian, China.
6. Kennedy, S., *price responsive electricity demand and wind power capacity value*, in *ASES annual conference proceedings, SOLAR*. 2006.
7. Botterud, A. and M. Korpas, *Modeling of power generation investment incentives under uncertainty in liberalized electricity markets.* p. 28.
8. (FERC), F.E.R.C., *Standerd Market Design and Structure, Notice of Proposed Rulemaking.* 2002.
9. ELEXON, *The Electricity Pool Guidance documents to the electricity pool.* 2009.
10. Sioshansi, F.P., *competitive electricity markets: design, implementation, performance* Frist ed. 2008, Oxford: Elsevier.
11. Schweppe, F.C., M.C. Caramanis, and R.D. Tabors, *evaluation of spot price based electricity rates.* IEEE transaction on Power apparatus and systems, 1985. **PAS-104**(7).
12. Bohn, R.E., M.C. Caramanis, and F.C. Schweppe, *Optimal pricing in electrical networks over space and time.* Rand Journal of Economics, 1984. **15**(3).
13. David, A.K. and Y.Z. Li, *consumer rationality assumptions in the real time pricing of electricity*, in *IEE international conference on advances in power system control, operation and management*. 1991: Hong Kong.
14. Xu, S. and W. Chen, *The reform of electricity power sector in the PR of China.* Energy Policy, 2005. **34**(2006).
15. FERC, *Assessment of demand response and advanced metering*, in *Staff report*. 2006. p. 228.
16. Burke, R.B. and M.I. Henderson, *incorporating demand response in operating reserve in New England.* 2005.
17. Burke, R.B., M.I. Henderson, and S.E. Widergren, *a look ahead at demand resoonse in New England.* 2008: p. 6.

18. Borenstein, S., M. Jaske, and A. Roseneld, *dynamic pricing, advanced metering and demand response in electricity markets*. 2002, university of California energy institute center for the study of energy markets. p. 103.
19. Ruff, L.E., *demand response: reality versus "resource".* the electricity journal, 2002.
20. Sergel, R., *NERC executive remarks demand response & reliability*. 2008.
21. Su, C.-L. and D. Kirschen, *Direct participation of demand-side in a pool-based electricity market.* power system technology, 2007. **Vol. 31**(20): p. 9.
22. Cramton, P., *electricity market design: the good, the bad, and the ugly*, in *Hawaii International Conference on System Sciences*. 2003.
23. Wu, T., et al., *pricing energy and ancillary services in integrated market systems by an optimal power flow.* IEEE transaction on power systems, 2004. **19**(1).
24. David, A.K. and Y.Q. He, *advances in global optimization of generation expansion planning.* IEE Proceeding Generation Transmission and Distribution, 1995. **142**(4).
25. Strbac, G., E.D. Farmer, and B.J. Cory, *Framework for the incorporation of demand-side in a competitive electricity market.* IEE Proceeding Generation Transmission and Distribution, 1996. **143**(3).
26. Shayesteh, E., et al., *Congestion management using demand response programs in power market.* 2008.
27. Motto, A.L., et al., *Network-constrained multiperiod auction for a pool-based electricity market.* Power engineering review, IEEE, 2002. **22**(6).
28. Goel, L., Q. Wu, and P. Wang, *Reliability enhancement of a deregulated power system considering demand response.* 2006.
29. Kirschen, D.S., et al., *Factoring the elasticity of demand in electricity prices.* IEEE transactions on power systems, 2000. **15**(2).
30. David, A.K. and Y.Z. Li, *effect of inter-temporal factors on the real time pricing of electricity.* IEEE transaction on power systems, 1993. **8**(1).
31. Lijesen, M.G., *the real-time price elasticity of electricity.* Energy Economics, 2007. **29**: p. 249-258.
32. Aalarni, H., G.R. Yousefi, and M.P. Moghadam, *demand response model considering EDRP and TOU programs.* 2008.
33. Daryanian, B., R.E. Bohn, and R.D. Tabors, *optimal demand-side response to electricity spot prices for storage-type customers.* IEEE transaction on power systems, 1989. **4**(3).
34. Schwarz, P.M., et al., *industrial response to electricity real-time prices: short run and long run.* Economic Inquiry, 2002. **40**(4): p. 597-610.
35. Roos, J.G. and I.E. Lane, *Industrial power demand response analysis for one-part real-time pricing.* IEEE transaction on power systems, 1998. **13**(1).
36. Reiss, P.C. and M.W. White, *Household electricity demand revisited.* review of economic studies, 2005. **(2005) 72**: p. 852-883.
37. Schweppe, F.C., et al., *Spot pricing of electricity.* 1988, Boston, MA: Kluwer Academic Publishers.